NYQUIST PLOTS AND STABILITY CRITERION

Topics Covered

- Nyquist Diagrams
- Nyquist Stability Analysis

Prerequisites

- Hardware Interfacing laboratory experiment
- Filtering laboratory experiment
- Stability Analysis laboratory experiment

1 Background

The gain and phase margins of a system are used to assess the stability of a system. The gain margin indicates how much open-loop gain is required before the system goes unstable. The phase margin indicates how much phase shift is needed in order to make the system go unstable. These are key indicators used to assess the stability of a system. The Nyquist diagram can be used to find these stability margins.

The block diagram of the closed-loop system with a generic compensator $C(s)$ and the open-loop plant transfer function $P(s)$ is shown in Figure 1.1. This will be used to control the angular rate of the QUBE-Servo 2.

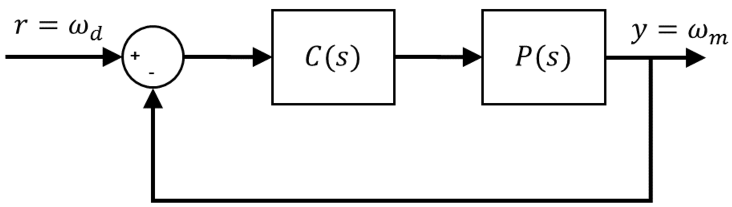


Figure 1.1: Generic closed-loop system

The forward loop transfer function is defined as:

$$L(s) = C(s)P(s) \tag{1.1}$$

Recall the first-order voltage to speed DC motor transfer function:

$$P(s) = \frac{\Omega_m(s)}{V_m(s)} = \frac{K}{\tau s + 1} \tag{1.2}$$

where K is the steady-state gain, τ the model time constant, and $\Omega_m(s) = \mathcal{L}\{\omega_m(t)\}$ is the speed of the motor (i.e. the inertial disc), and $V_m(s) = \mathcal{L}\{v_m(t)\}$ is the applied motor voltage.

Figure 1.2 represents the Nyquist curve of the loop transfer function and how it can be used to assess the gain and phase margins.

The gain margin is defined as the increase in the system gain when the phase is $-180°$ that will result in a marginally stable system. In terms of a Nyquist plot, a phase of $-180°$ indicates that the system's trajectory intersects the negative real axis. It is clear from the Nyquist stability criterion that the system is marginally stable for an intersection of the point $(-1, 0)$. Given that the current intersect with the negative real axis is $1/g_m$ as in Figure 1.2, it is clear that a gain of g_m is necessary to reach $(-1, 0)$. The gain margin is often defined in terms of the logarithmic decibel measure as

$$GM \text{ [dB]} = -20 \log \left(\frac{1}{g_m} \right) \text{ dB} = 20 \log \left(g_m \right) \text{ dB}. \tag{1.3}$$

The phase margin ϕ_m is defined as the angle through which the locus the loop transfer function $L(s)$ must be rotated so that the unity magnitude point is $(-1, 0)$, i.e. the system is marginally stable. The gain crossover frequency ω_{gc} is the frequency where the magnitude of the loop transfer function equals one: $|L(j\omega_{gc})| = 1$. At the gain crossover frequency, the phase margin is defined as the distance of the system phase angle above $-180°$.

The delay margin, T_{dm}, is another stability concept which is defined as the amount of time delay that brings the system to the stability boundary. For a simple system where the Nyquist curve is as shown in Figure 1.2, the delay margin is given by

$$T_{dm} = \frac{\phi_m}{\omega_{gc}} \tag{1.4}$$

where ϕ_m is the phase margin and the ω_{gc} is the gain crossover frequency. The actual closed-loop implementation that is used to assess the stability margins of the QUBE-Servo 2 is shown in Figure 1.3.

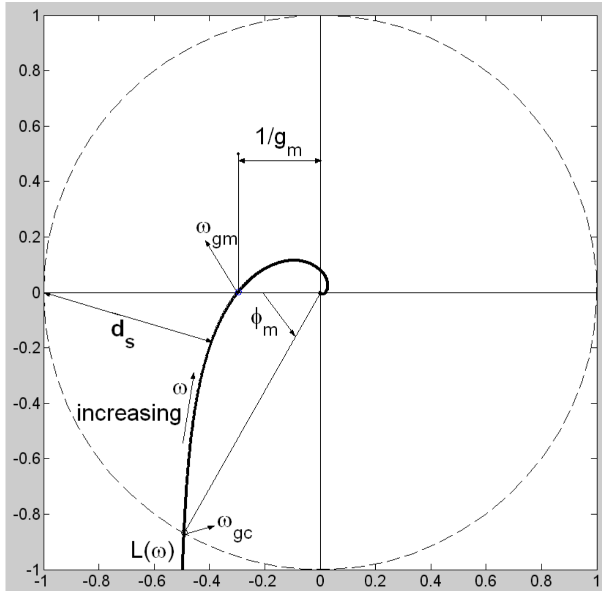


Figure 1.2: Nyquist diagram

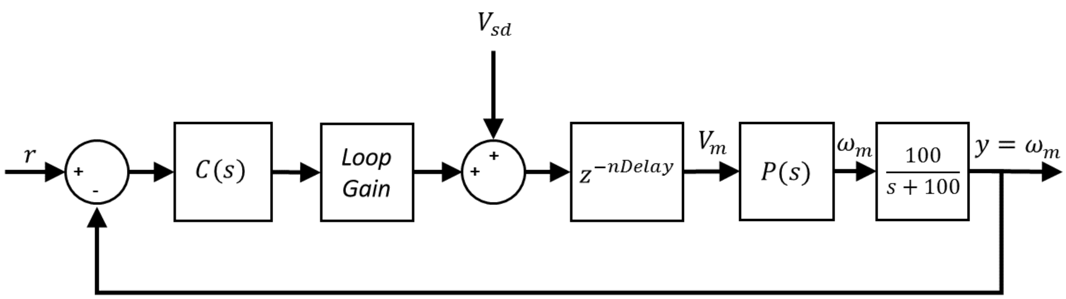


Figure 1.3: Closed-loop implementation used to find stability margins experimentally

The *Loop Gain* parameter is a pure gain and is used to estimate the system gain margin. The *nDelay* parameter is an integer number of sample periods and is used to determine the extra time delay needed to make the system unstable, where z^{-1} is the discrete time operator. A disturbance torque can also be simulated by applying an additional voltage, V_{sd}, to the motor input (through the software).

In this lab, a PI control will be used to control the speed:

$$C(s) = k_p + \frac{k_i}{s} \tag{1.5}$$

The plant transfer $P(s)$ function that will be used for the analysis includes the low-pass filter $100/(s+100)$ that is used to filter out the angular velocity noise as well as the time delay introduced by the control loop rate. Thus the plant transfer being used is the following:

$$P(s) = \frac{100}{s+100}\frac{K}{\tau s+1}e^{-hs} \tag{1.6}$$

where h is the sampling/control loop interval of the controller (e.g. control loop frequency of 100 Hz equal a control rate of $h = 0.01$ s). Including the effect of filtering and sampling delay will improve the accuracy of the stability analysis.

2 In-Lab Exercises

2.1 Assess Stability Margins using Nyquist Simulation

1. In MATLAB®, design a script that plots the Nyquist diagram of the system shown in Figure 1.1 using the plant transfer function given in Equation 1.2 for $P(s)$ and the PI controller given in Equation 1.5 for $C(s)$. To do this, open the script called `qube_nyquist_student.m`. The script automatically calculates the PI control gains based on second-order specifications ω_n and τ, similarly as done in Stability Analysis laboratory experiment for position control. It then constructs the loop transfer function using the open-loop representation in Equation 1.6. Set the MATLAB variables *K* and *tau* to the model parameters of the QUBE-Servo 2.

   Hint: To conduct the most accurate stability analysis, it is recommended to run one of the modeling labs, e.g. Step Response Modeling laboratory experiment, on the QUBE-Servo 2 you are using to obtain its model parameters, K and τ.

2. Run the script to generate the Nyquist plot of the loop transfer function: $L(s) = P(s)C(s)$.

   Note: This script plots the Nyquist using manual commands, it does not use the standard MATLAB *Nyquist* command.

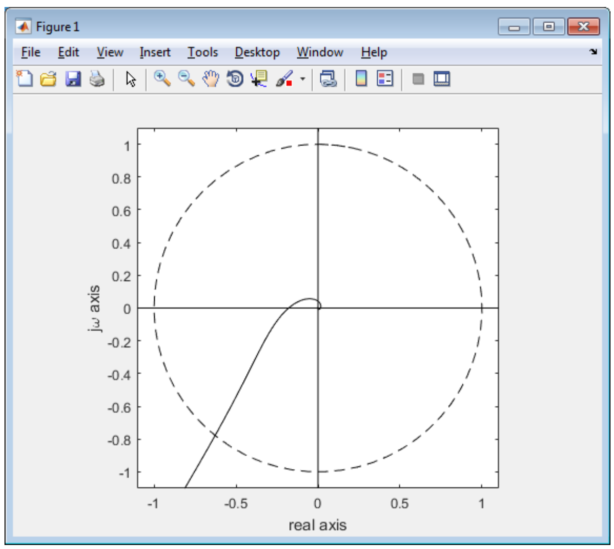


Figure 2.1: Example Nyquist plot of QUBE-Servo 2

3. Find the gain margin, g_m, using the Nyquist plot. Find the gain margin, g_m, and gain margin frequency, ω_{gm}, using the Nyquist diagram and data. Show your calculations and MATLAB script commands used.

   Hint: Use the `find` or `while` loop MATLAB commands.

4. Use MATLAB commands in the script (or in a separate script) that finds the phase margin and gain crossover frequency. Also, assess the delay margin given a control loop rate of $h = 0.01$ s. Use the labels in Figure 1.2 and the MATLAB variables: `RealL`, `ImagL`, `MagL`, and `w`. Show your calculations and MATLAB script commands used.

2.2 Find Stability Margins Experimentally

The `q_qube2_stability_margins` Simulink model shown in Figure 2.2 implements the closed-loop diagram shown in Figure 1.3 and is used to evaluate the stability margins of the QUBE-Servo 2. The motor speed is controlled using a PI compensator. The Loop Gain is a pure gain and is used to find the gain margin. The nDelay parameter is used to determine the extra time delay needed to make the system unstable (i.e. the delay margin). A disturbance torque can be simulated by toggling the Disturbance Switch to apply a 1 V disturbance voltage to the motor.

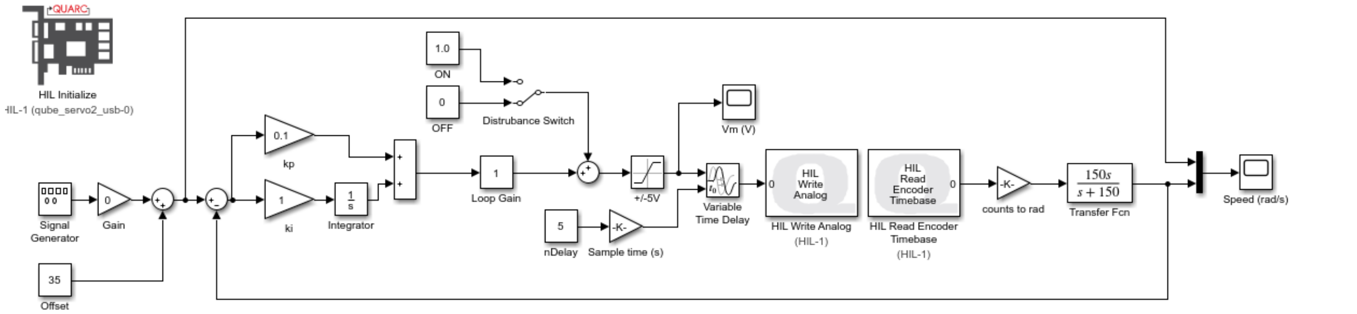
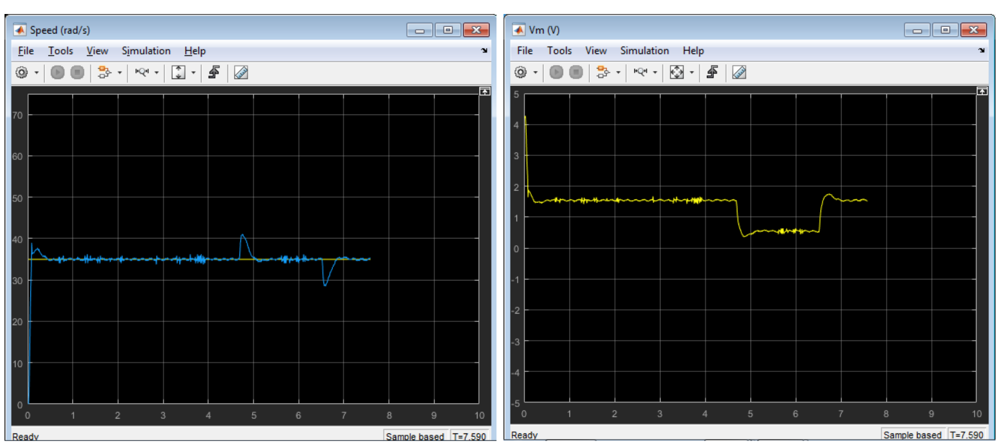


Figure 2.2: PI speed control with Loop Gain and Sample Delay disturbances

Note: The *HIL Read Encoder timebase* block is used for the controller to be triggered by the clock on the data acquisition device. Using hardware-based timing is recommended when doing speed control that derives motor speed computationally from a position sensor (i.e. encoder).

1. Open the `q_qube2_stability_margins` Simulink model shown in Figure 2.2. Alternatively, the model can be designed yourself using the QUARC controller created in QUBE-Servo 2 Stability Analysis laboratory experiment.
2. Configure the setpoint to be a constant of 35.0 rad/s. To do this, set the Gain block connected to the Signal Generator to 0 and the Offset block to 35.
3. Set the kp and ki Simulink Gain blocks to the PI gains found in the Simulate section using the `qube_nyquist_student.m` script.
4. Build and run the QUARC controller. Figure 2.3 shows a sample response when the controller is started using the default gains of kp = 0.1 and ki = 1 and applying a disturbance at the 4.5 sec mark.



(a) Motor Speed (b) Motor Voltage

Figure 2.3: QUBE-Servo 2 response with a Loop Gain of 1 and nDelay of 0

5. The gain margin value g_m corresponds to the value of the extra pure gain in the closed-loop necessary to make the system on the verge of instability. Therefore to experimentally determine the gain margin, increase

the Loop Gain by increments of 0.1 (starting from 1) and ensure the nDelay is set to 0. At every iteration, apply the 1V disturbance using the Disturbance Switch. Once a disturbance is applied that causes the output speed to oscillate without decaying, the margin has been reached. At this point, stop the QUARC controller and record the corresponding *Loop Gain* used. The frequency of the oscillations obtained is the gain-margin frequency ω_{gm}. Show the obtained response and find the corresponding gain-margin frequency.

6. As expressed in Equation 1.4, the time delay is a measure of the phase margin. *nDelay* represents the extra sample time delay (equivalent to an extra phase lag) required to make the system on the verge of instability. To assess the delay margin, increase the *nDelay* parameter starting from 0 by steps of 1 and ensure that Loop Gain is set to 1. At every iteration, apply a 1V disturbance using the Disturbance Switch. Once the output speed oscillations do not decay, or the speed becomes unstable, you have reached the margin. At this point, stop the QUARC controller and record the corresponding *nDelay*. The frequency of the oscillations is the response is the gain crossover frequency ω_{gc}. Attach the obtained response and calculate the corresponding gain crossover frequency.

7. Summarize the gain margin, gain-margin frequency, delay margin, and gain-crossover frequency obtained theoretically using Nyquist in 2.1with the experimental ones. How do the experimental results compare with the ones obtained using the Nyquist method? Given a reason why there could be a difference between the results.

8. Make sure the QUARC controller is stopped.

9. Power off the QUBE-Servo 2 if no additional experiments will be performed.

Quanser Inc.
119 Spy Court
Markham, Ontario
L3R 5H6
Canada
info@quanser.com
Phone: 1-905-940-3575
Fax: 1-905-940-3576

Printed in Markham, Ontario.

For more information on the solutions Quanser Inc. offers, please visit the web site at:
http://www.quanser.com