STATE-SPACE MODELING

Topics Covered

- Modeling from first-principles.
- State-space representation.
- Model validation.

Prerequisites

- Hardware Interfacing laboratory experiment.
- Filtering laboratory experiment.

1 Background

1.1 Linear State-Space Representation

The standard state-space representation of a multi-input multi-output (MIMO) continuous linear-time invariant (LTI) system with n state variables, r input variables, and m output variables is

$$\dot{x}(t) = Ax + Bu \tag{1.1}$$

$$y(t) = Cx(t) + Du(t) \tag{1.2}$$

where x is the vector of state variables $(n \times 1)$, u is the control input vector $(r \times 1)$, y is the output vector $(m \times 1)$, A is the system matrix $(n \times n)$, B is the input matrix $(n \times r)$, C is the output matrix $(m \times n)$, and D is the feed-forward matrix $(m \times r)$.

The block diagram representation of state-space Equation 1.1 and Equation 1.2 is shown in Figure Figure 1.1.

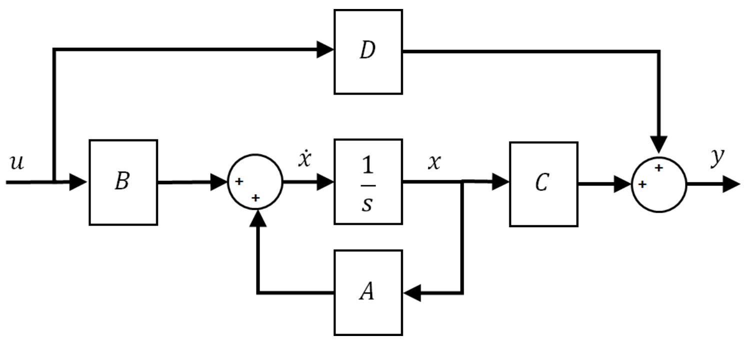


Figure 1.1: State-Space Block Diagram

1.2 DC Motor Modeling

This section summarizes how to find the equations of motion (EOMs) of the DC motor. The motor electrical equation is

$$v_m(t) - R_m i_m(t) - k_m \dot{\theta}_m(t) = 0 \tag{1.3}$$

where $v_m(t)$ is the motor input voltage (the control input), R_m is the motor electrical resistance, $i_m(t)$ is the current, k_m is the back-emf constant, and $\theta_m(t)$ is the angular position of the motor shaft (i.e. the inertia disc).

The motor shaft equation is expressed as

$$J_{eq}\ddot{\theta}(t) = \tau_m(t) \tag{1.4}$$

where J_{eq} is the total or equivalent moment of inertia acting on the motor shaft and τ_m is the applied torque from the DC motor. Based on the current applied, the torque is

$$\tau_m(t) = k_t i_m(t) \tag{1.5}$$

where k_t is the motor current torque constant.

See Block Diagram Modeling laboratory experiment for more information about finding the servo equations of motion.

2 In-Lab Exercises

The SIMULINK® model shown in Figure 2.1 applies a 1 V step to the QUBE-Servo 2 hardware, using QUARC®, and the state-space model of the servo. The measured and simulated speed response (i.e. from the model) are plotted in the same scope to compare.

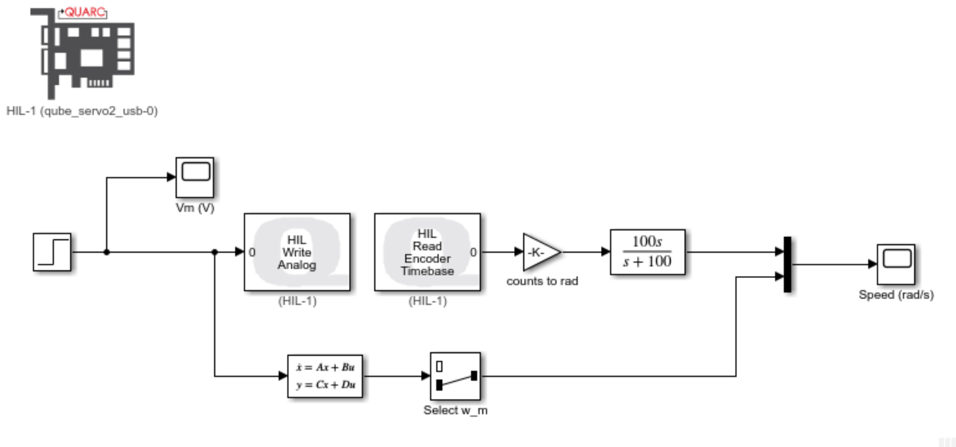
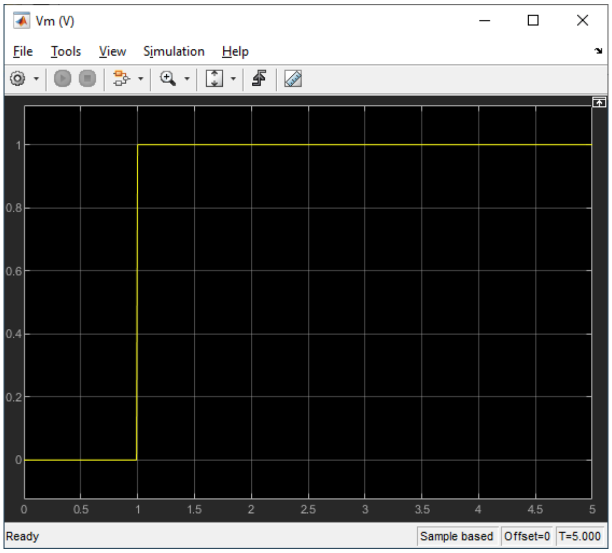
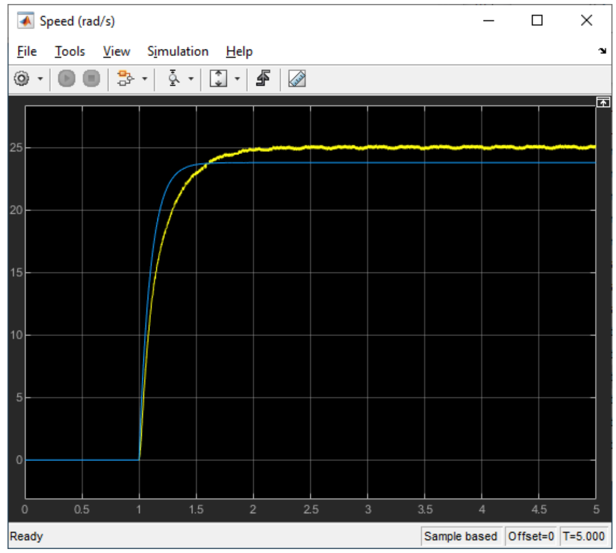


Figure 2.1: Applies a step voltage and displays measured and simulated DC motor response.

1. Formulate the differential equation relating the motor position, θ_m, and the motor input voltage, $v_m(t)$, using Equation 1.3, Equation 1.4, and Equation 1.5.
2. Derive the state-space model of the DC motor from the differential equation you obtained in Exercise 1 for the following state variables: $x_1 = \theta_m(t)$ and $x_2 = \dot{\theta}_m(t)$, $y_1 = \theta_m(t)$ and $y_2 = \dot{\theta}_m(t)$ (i.e. measuring motor position and speed), and the input variable $u = v_m(t)$.
3. Based on the state space model derived in Step 2 and using the parameters defined in the the MATLAB® script `qube2_param.m` provided, create the MATLAB® script that constructs a MATLAB state-space model and generate its step response.
4. Open or design the Simulink model shown in Figure 2.1. This applies a 1 V step to the QUBE-Servo 2 system and its state-space model. Use the model already designed in the Filtering laboratory experiment if Figure 2.1 is to be designed.
5. Run the MATLAB® script created in Exercise 3 to load the state space model parameters in the MATLAB® workspace. Ensure that the generated matrices match your solution in Step 2.
6. Build and run the model. The scope response should be similar to Figure 2.2. Attach a screen capture of your scopes. Does your model represent the actual DC motor well?



(a) Input Voltage (V)



(b) Motor Angular Rate (rad/s)

Figure 2.2: Step response of the DC motor and State-Space Model

7. Stop the QUARC® controller.

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