PROPORTIONAL CONTROL

Topics Covered

- Servo position control.
- Servo speed control.
- Steady-state error.
- Proportional compensator.

Prerequisites

- Hardware Interfacing laboratory experiment.
- Filtering laboratory experiment.
- Stability Analysis laboratory experiment.

1 Background

1.1 Servo Model

The QUBE-Servo 2 voltage-to-position transfer function is

$$P_{pos}(s) = \frac{\Theta_m(s)}{V_m(s)} = \frac{K}{s(\tau s + 1)}, \tag{1.1}$$

and the QUBE-Servo 2 voltage-to-velocity transfer function is

$$P_{vel}(s) = \frac{\Omega_m(s)}{V_m(s)} = \frac{K}{\tau s + 1}, \tag{1.2}$$

where $K = 23.2$rad/s/V is the model steady-state gain, $\tau = 0.13$s is the model time constant, $\Theta_m(s) = \mathcal{L}\left[\theta_m(t)\right]$ is the motor / disk position, $\Omega_m(s) = \mathcal{L}\left[\omega_m(t)\right]$ is the motor / disk velocity and $V_m(s) = \mathcal{L}\left[v_m(t)\right]$ is the applied motor voltage. It is recommended to find more precise model parameters, K and τ, for your particular servo (e.g., performing the Step Response Modeling lab).

1.2 PID Control

The proportional, integral, and derivative control can be expressed mathematically as follows

$$u(t) = k_p e(t) + k_i \int_0^t e(\tau)d\tau + k_d \frac{de(t)}{dt}. \tag{1.3}$$

The corresponding block diagram is given in Figure 1.1. The control action is a sum of three terms referred to as proportional (P), integral (I) and derivative (D) control gain. The controller Equation 1.3 can also be described by the transfer function

$$C(s) = k_p + \frac{k_i}{s} + k_d s. \tag{1.4}$$

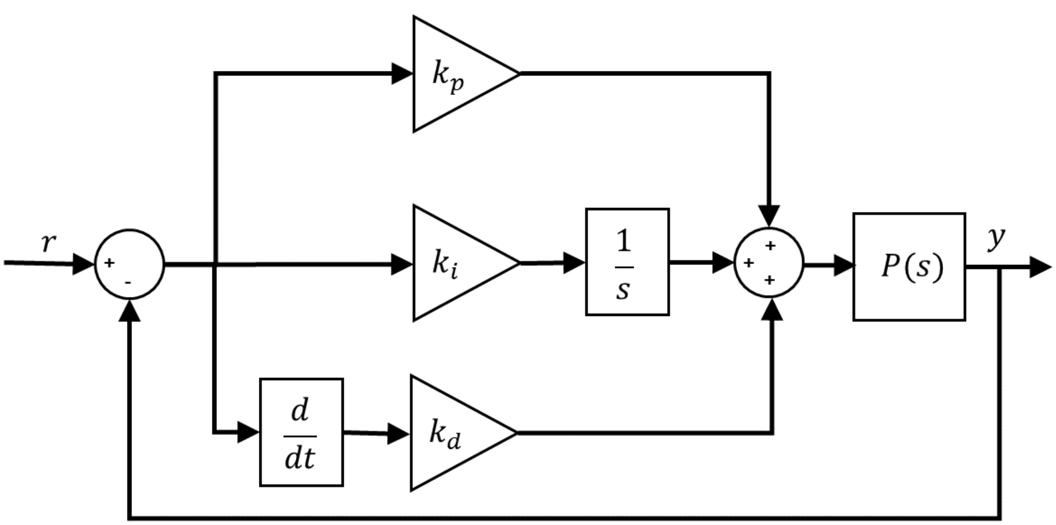


Figure 1.1: Block diagram of PID control

The functionality of the PID controller can be summarized as follows. The proportional term is based on the present error, the integral term depends on past errors, and the derivative term is a prediction of future errors.

1.3 Proportional Position Control

In this lab, we will examine how a pure proportional controller can be used to control the QUBE-Servo 2 position, see Figure 1.2.

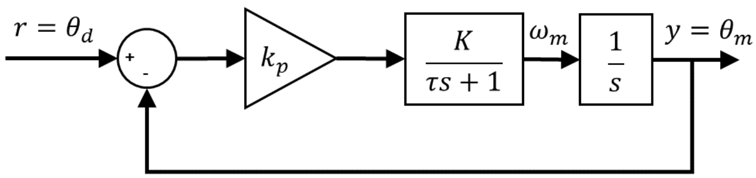


Figure 1.2: Proportional control of servo position

The proportional (P) control has the following structure

$$u(t) = k_p\left(r(t) - y(t)\right), \tag{1.5}$$

where k_p is the proportional gain, $r(t) = \theta_d(t)$ is the setpoint or reference motor / load angle for the position control, $y(t) = \theta_m(t)$ is the measured load shaft angle, and $u(t) = v_m(t)$ is the control input (applied motor voltage).

The closed-loop transfer function of the QUBE-Servo 2 is denoted $Y(s)/R(s) = \Theta_m(s)/\Theta_d(s)$. Assume all initial conditions are zero, i.e. $\theta_m(0^-) = 0$ and $\dot{\theta}_m(0^-) = 0$, taking the Laplace transform of Equation 1.5 yields

$$U(s) = k_p\left(R(s) - Y(s)\right),$$

which can be substituted into Equation 1.1 to yield

$$Y(s) = \frac{K}{s(\tau s + 1)}(k_p(R(s) - Y(s)).$$

Solving for $Y(s)/R(s)$, we obtain the closed-loop expression

$$\frac{Y(s)}{R(s)} = \frac{Kk_p}{\tau s^2 + s + Kk_p}. \tag{1.6}$$

1.4 Proportional Speed Control

Similar to the proportional position control described above, the proportional speed control has the same structure as Equation 1.5.

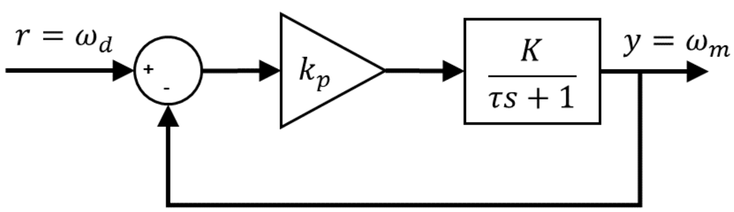


Figure 1.3: Proportional control of servo speed

As shown by the block diagram in Figure 1.3, here $r(t) = \omega_d(t)$ is the reference angular velocity, $y(t) = \omega_m(t)$ is the measured angular velocity, and $u(t) = v_m(t)$ is the control input (applied motor voltage).

Under similar assumptions as above and with Equation 1.2, the following closed-loop transfer function $Y(s)/R(s)$ can be obtained:

$$\frac{Y(s)}{R(s)} = \frac{Kk_p}{\tau s + 1 + Kk_p}. \tag{1.7}$$

1.5 Final Value Theorem

The Final Value Theorem can be used to determine the steady-state or final value of the system output $y(t)$ given its Laplace transform $Y(s)$. For a system with an unstable pole (i.e. a pole in the right half of the s-plane), the final value is unbounded. If the system is marginally stable with a complex conjugate pole pair on the imaginary axis, the output of the system will be oscillatory and the final value is not defined. For a stable system response (i.e. all poles of the system are strictly in the left half of the s-plane), the following holds

$$\lim_{s \to 0} sY(s) = \lim_{t \to \infty} y(t). \tag{1.8}$$

1.6 Steady State Error

Steady-state error is the difference between the reference input and output signals after the system response has settled. Thus, for a time t when the system is in steady-state, the steady-state error equals

$$e_{ss} = r_{ss}(t) - y_{ss}(t) \tag{1.9}$$

where $r_{ss}(t)$ is the value of the steady-state input and $y_{ss}(t)$ is the steady-state value of the output.

We can find the error transfer function $E(s)$ in Figure 1.2 in terms of the reference $R(s)$, the plant $P(s)$, and the compensator $C(s)$. The Laplace transform of the error is

$$E(s) = R(s) - Y(s) \tag{1.10}$$

2 In-Lab Exercises

2.1 Proportional Position Control

The Simulink model shown in Figure 2.1 implements the proportional position controller describe in 1.3.

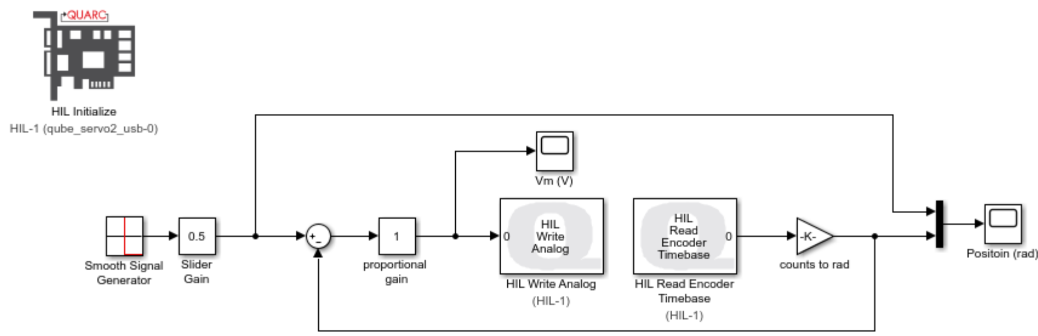


Figure 2.1: Simulink/QUARC model implementing a proportional position control on QUBE-Servo 2

1. The voltage that can be applied to the QUBE-Servo 2 is limited to ± 10V. Determine the maximum proportional gain for the square wave reference signal of ± 0.5rad above that does not saturate the QUBE-Servo 2.
2. Is it possible to get a desired second-order system response for any ω_n and ζ by only using proportional feedback gain only?

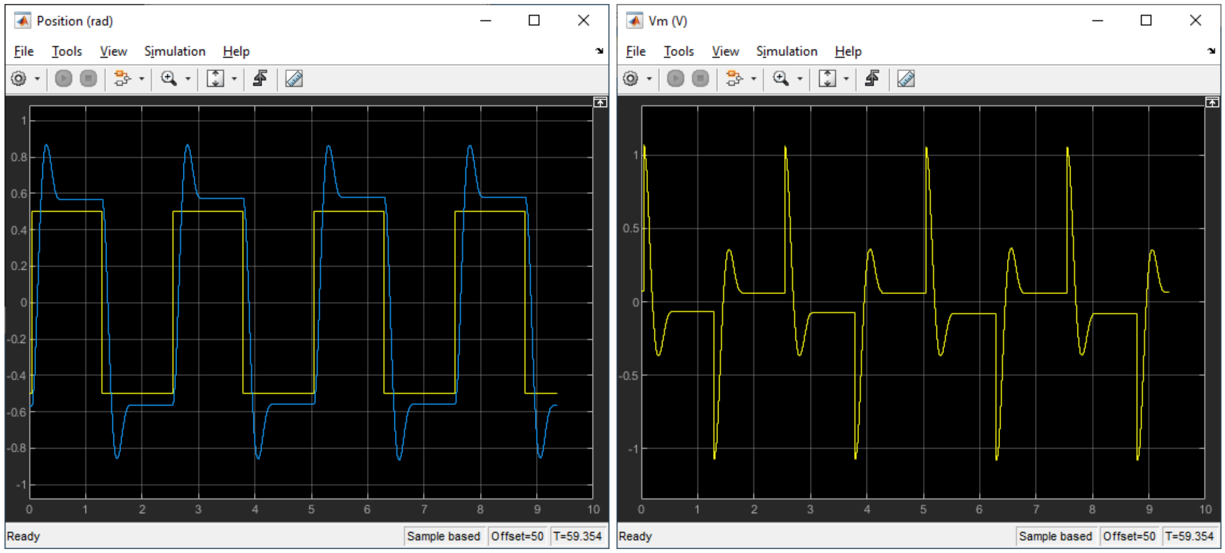
   Hint: Compare the QUBE-Servo 2 closed-loop transfer function given in Equation 1.6 to the standard second-order system

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2}.$$

3. Open the Simulink diagram shown in Figure 2.1 (if supplied) or design your own.
4. Set the Signal Generator block such that the servo command (i.e. reference angle) is a square wave with an amplitude of 0.5rad and at a frequency of 0.4Hz.
5. Build and run the QUARC controller. The response should look similar to Figure 2.2 when the proportional gain is set to 1.
6. Run the controller with a proportional gain of 1.5. Attach the position response and motor voltage scope responses.
7. Measure the percent overshoot and peak time of the response when $k_p = 1.5$.
8. Is there a steady-state error? If so, evaluate it.
9. Vary k_p between 1 and 5. How does the proportional gain affect the servo position control response?
10. Vary k_p between 0.1 and 1. What happens when k_p is decreased?
11. Describe the response for very small gains of k_p (i.e. below 0.5). Does the system respond as expected? Explain.

    Hint: If the system's response does not meet your expectations, verify that the desired control signals are applied to the QUBE-Servo 2.

12. Stop the QUARC controller.



(a) Position (b) Voltage

Figure 2.2: Proportional position control response with $k_p = 1$.

2.2 Proportional Speed Control

The Simulink diagram shown in Figure 2.3 implements the proportional speed control shown in Figure 1.2 on the QUBE-Servo 2.

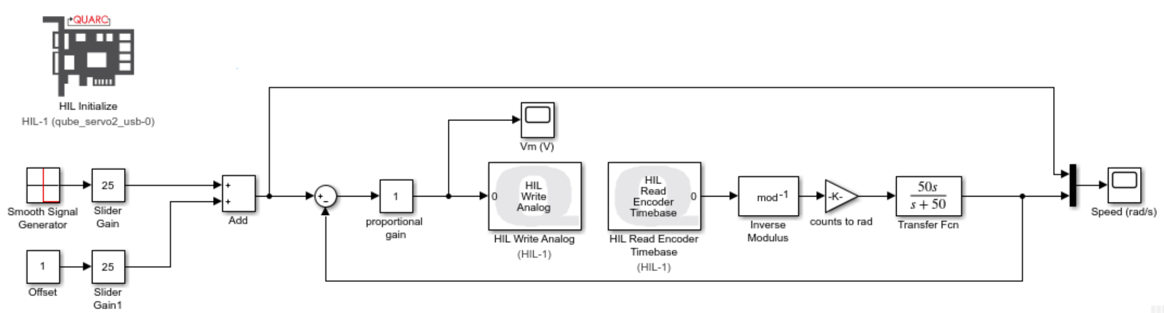


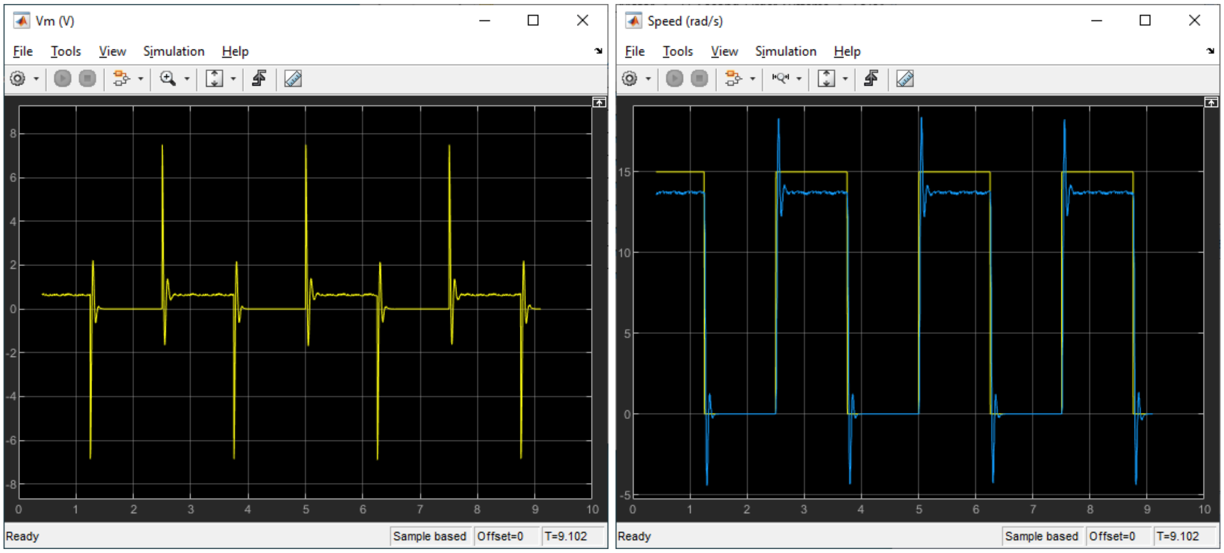
Figure 2.3: Simulink/QUARC model implementing proportional speed control on QUBE-Servo 2

1. Find the closed-loop transfer function $E(s)/R(s)$ in Figure 1.3 that represents the dynamics between a desired angular rate, $R(s) = \Omega_d(s)$, and the error, $E(s) = R(s) - Y(s) = \Omega_d(s) - \Omega_m(s)$.
2. Find the steady-state error of the system, e_{ss}, for the reference step input $R(s) = R_0/s$ where R_0 is the desired angular rate step amplitude.

   Hint: Use the Final-Value Theorem.

3. Evaluate the steady-state error if the proportional gain is 0.5Vs/rad and a step amplitude of 15rad/s. Use either the default model parameters given in Section 1.1 or, for more accurate results, find K for the QUBE-Servo 2 being used by going through one of the modeling laboratories, e.g. Step Response Modeling laboratory experiment.
4. Open the Simulink diagram shown in Figure 2.3 (if supplied) or design your own.
5. To generate a reference speed command with an amplitude of 15rad/s, set the Amplitude and Offset gain blocks both to 7.5rad/s and ensure the Smooth Signal Generator block is configured to output a square wave at a frequency of 0.4Hz.

6. Build and run the QUARC controller.
7. The voltage and speed scope should look similar to the response shown in Figure 2.4. Attach the speed response of the servo you obtained.



(a) Voltage

(b) Speed

Figure 2.4: Steady-state error with 15rad/s amplitude step and $k_p = 0.5$

8. Measure the steady-state error. How does it compare with your value computed above? If they do not match, give one reason why there would be a difference.

   Hint: Use the Cursor Measurement tool in the Simulink Scope to take your measurements.

9. Show how the error can be decreased by half its current magnitude. Validate your results with the QUBE-Servo 2 and show your response.

   Hint: Recall how the steady state error was defined and identify which parameters you can adjust.

10. Stop the QUARC® controller.
11. Power off the QUBE-Servo 2 if no more experiments will be conducted in this session.

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