MOMENT OF INERTIA

Topics Covered

- Finding moment of inertia analytically and experimentally.

Prerequisites

- Hardware Interfacing laboratory experiment

1 Background

The free-body diagram of the QUBE-Servo 2 pendulum system is shown in Figure 1.1.

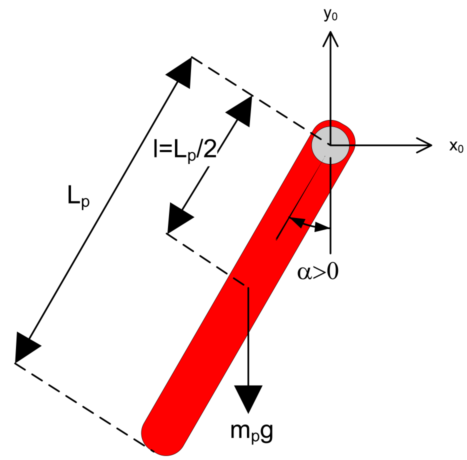

Figure 1.1: Free-body diagram of pendulum

From the free-body diagram in Figure 1.1, the resulting nonlinear equation of motion of the pendulum is

$$J_p \ddot{\alpha}(t) = m_p g l \sin(\alpha(t)), \tag{1.1}$$

where J_p is the moment of inertia of the pendulum at the pivot axis, m_p is the mass of the pendulum, L_p is the length of the pendulum (from pivot to end), and $l = L_p/2$ is the distance between the pivot and center of mass.

The moment of inertia of the pendulum can be found experimentally. Assuming the pendulum is not actuated, linearizing Equation 1.1 and solving for the differential equation yields

$$J_p = \frac{m_p g l}{(2\pi f)^2}, \tag{1.2}$$

where f is the measured frequency of the pendulum as the arm remains rigid. The frequency is calculated using

$$f = \frac{n_{cyc}}{\Delta t} \tag{1.3}$$

where n_{cyc} is the number of cycles and Δt is the duration of these cycles. Alternatively, J_p can be calculated using the moment of inertia expression

$$J = \int r^2 dm, \tag{1.4}$$

where r is the perpendicular distance between the element mass dm and the axis of rotation.

2 In-Lab Exercises

Based on the model already designed in Hardware Interfacing laboratory experiment, design a model that measures the pendulum angle using the encoder as shown in Figure 2.1.

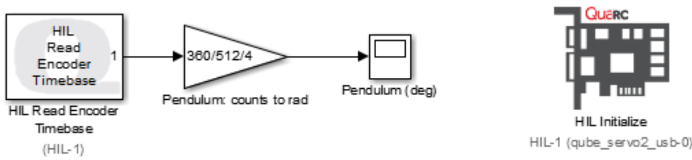

Figure 2.1: Displays measured pendulum angle

1. Find the moment of inertia acting about the pendulum pivot using the free-body diagram. Make sure you evaluate it numerically using the parameters defined in the QUBE-Servo 2 User Manual.

   Hint: For solid objects with a uniform density, you can express the differential mass in terms of differential length.

2. Build the SIMULINK® diagram shown in Figure 2.1. Enter $360/(512 * 4)$ in the encoder sensor gain to measure the pendulum angle in degrees.

3. Build and run the QUARC® controller. With the controller running, manually perturb the pendulum while holding the rotary arm in place. The scope response should be similar to Figure 2.2.

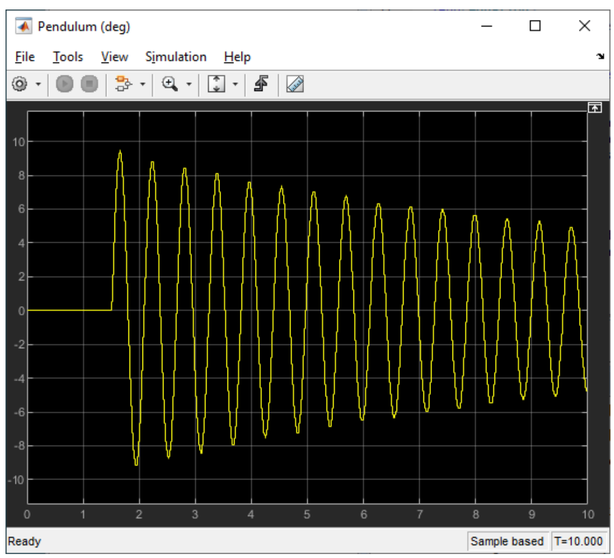

Figure 2.2: Free-oscillation response of pendulum

4. Find the frequency and moment of inertia of the pendulum using the observed results.

5. Compare the moment of inertia calculated analytically in Exercise 1 and the moment of inertia found experimentally. Is there a large discrepancy between them?

Quanser Inc.
119 Spy Court
Markham, Ontario
L3R 5H6
Canada
info@quanser.com
Phone: 1-905-940-3575
Fax: 1-905-940-3576

Printed in Markham, Ontario.

For more information on the solutions Quanser Inc. offers, please visit the web site at:
http://www.quanser.com