STEP RESPONSE MODELING

Topics Covered

- First order transfer functions.
- Obtaining the QUBE-Servo 2 model experimentally using step response (i.e. bump test method).
- Model validation.

Prerequisites

- Hardware Interfacing laboratory experiment.
- Filtering laboratory experiment.

1 Background

Step response modeling, also known as the *bump test* , is a simple test based on the step response of a stable system. A step input is given to the system and its response is recorded. As an example, consider a system given by the following transfer function:

$$\frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1} \tag{1.1}$$

The step response shown in Figure 1.1 is generated using this transfer function with $K = 5$ rad/V.s and $\tau = 0.05$ s.

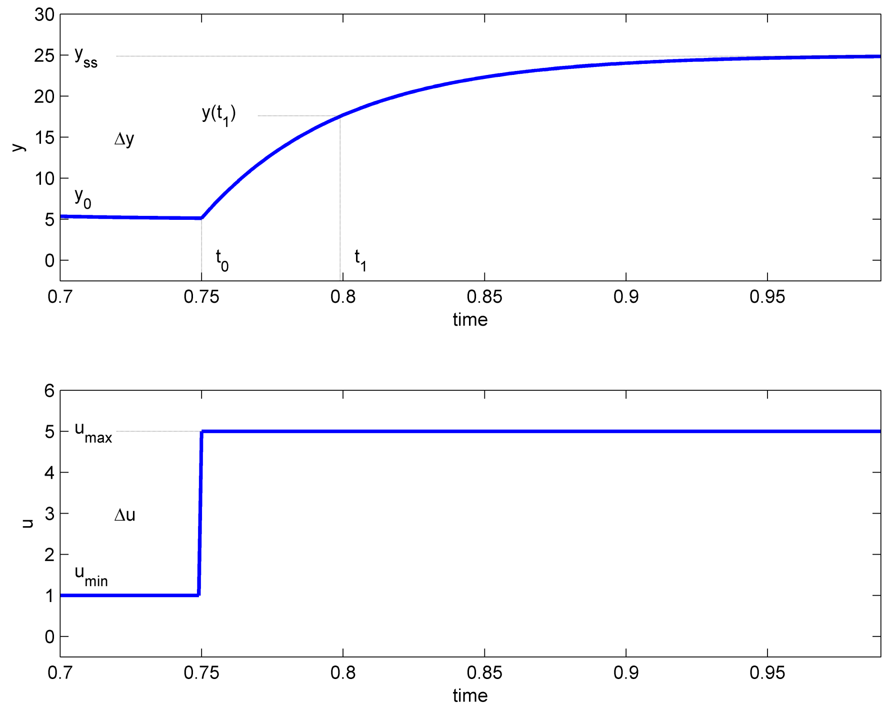


Figure 1.1: Input and output signal used in the bump test method

The step input begins at time t_0. The input signal has a minimum value of u_{min} and a maximum value of u_{max}. The resulting output signal is initially at y_0. Once the step is applied, the output tries to follow it and eventually settles at its steady-state value y_{ss}. From the output and input signals, the steady-state gain is

$$K = \frac{\Delta y}{\Delta u} \tag{1.2}$$

where $\Delta y = y_{ss} - y_0$ and $\Delta u = u_{max} - u_{min}$. The time constant of a system τ is defined as the time it takes the system to respond to the application of a step input to reach $1 - 1/e \approx 63.2\%$ of its steady-state value, i.e. for Figure 1.1

$$t_1 = t_0 + \tau,$$

where

$$y(t_1) = 0.632\Delta y + y_0. \tag{1.3}$$

Then, we can read the time t_1 that corresponds to $y(t_1)$ from the response data in Figure 1.1. From this, the model time constant can be found as:

$$\tau = t_1 - t_0. \tag{1.4}$$

1.1 Applying this to the QUBE-Servo 2

Going back to the QUBE-Servo 2 system, the s-domain representation of a step input voltage with a time delay t_0 is given by

$$V_m(s) = \frac{A_v e^{(-st_0)}}{s}, \tag{1.5}$$

where A_v is the amplitude of the step and t_0 is the step time (i.e. the delay).

The voltage $V_m(s)$ to speed $\Omega_m(s)$ transfer function (derived later in the Parameter Estimation laboratory experiment) is:

$$\frac{\Omega_m(s)}{V_m(s)} = \frac{K}{\tau s + 1} \tag{1.6}$$

where K is the model steady-state gain, τ is the model time constant, $\Omega_m(s) = \mathcal{L}\left[\omega_m(t)\right]$ is the load gear rate, and $V_m(s) = \mathcal{L}\left[v_m(t)\right]$ is the applied motor voltage.

If we substitute input Equation 1.5 into the system transfer function Equation 1.6, we get:

$$\Omega_m(s) = \frac{K A_v e^{(-st_0)}}{(\tau s + 1)s}. \tag{1.7}$$

We can then find the QUBE-Servo 2 motor speed step response in the time domain $\omega_m(t)$ by taking inverse Laplace of this equation

$$\omega_m(t) = K A_v \left(1 - e^{\left(-\frac{t-t_0}{\tau}\right)}\right) + \omega_m(t_0), \tag{1.8}$$

noting the initial conditions $\omega_m(0^-) = \omega_m(t_0)$.

2 In-Lab Exercises

Based on the models already designed in QUBE-Servo 2 Hardware Interfacing and Filtering laboratory experiment, design a model that applies a step of 2 V to the motor and reads the servo velocity using the encoder as shown in Figure 2.1.

To apply your step for a certain duration (e.g. 2.5 s), set the *Simulation stop time* of the SIMULINK® model. Using the saved response, the model parameters can then be found as discussed in Background section of this lab. For information on saving data to MATLAB for offline analysis, see the QUARC® help documentation (under *QUARC Targets | User's Guide | QUARC Basics | Data Collection*).

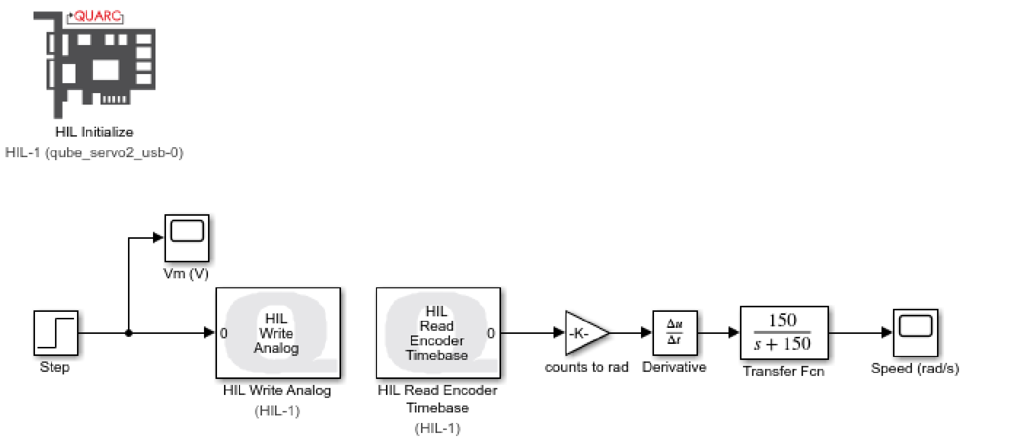
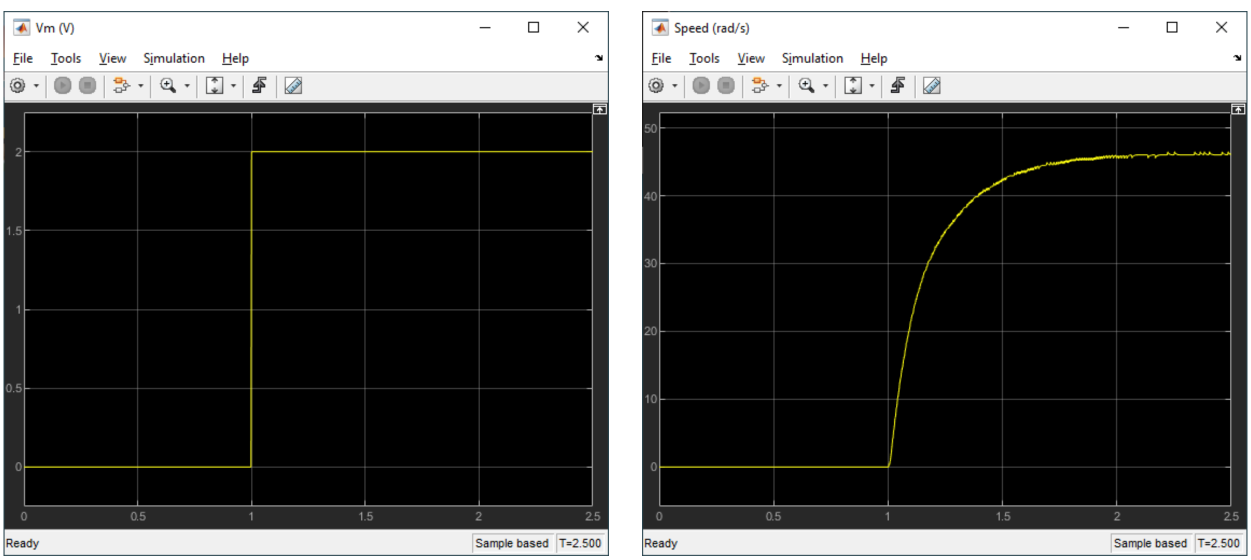


Figure 2.1: Applies a step voltage and measures corresponding servo speed

1. Run the QUARC controller to apply a 2 V step to the servo. The scope response should be similar to Figure 2.2.



(a) Motor Voltage

(b) Load Speed

Figure 2.2: Sample QUBE-Servo 2 step response

2. Plot the response in MATLAB® figure. For example, you can setup the scopes to save the measured load/disk speed and motor voltage to the MATLAB® workspace in the variables `data_wm` and `data_vm`, where the `data_wm(:,1)` is the time vector and `data_wm(:,2)` is the measured speed.

3. Find the steady-state gain using the measured step response.

   Hint: Use the *Cursor Measurements* tool in the SIMULINK® Scope or the *Data Tips* tool in the MATLAB® figure to take measurements directly from the response plots.

4. Find the time constant from the obtained response.

5. To check if your derived model parameters K and τ are correct, modify the Simulink diagram to include a Transfer Fcn block with the first-order model in Equation 1.1, as shown in Figure 2.3. Connect both the measured and simulated QUBE-Servo 2 responses to the scope using a Mux block (from the *Signal Routing* category). Build and run your QUARC® controller. Attach a MATLAB® figure displaying both the measured and simulated response in one plot, as well as in the input voltage.

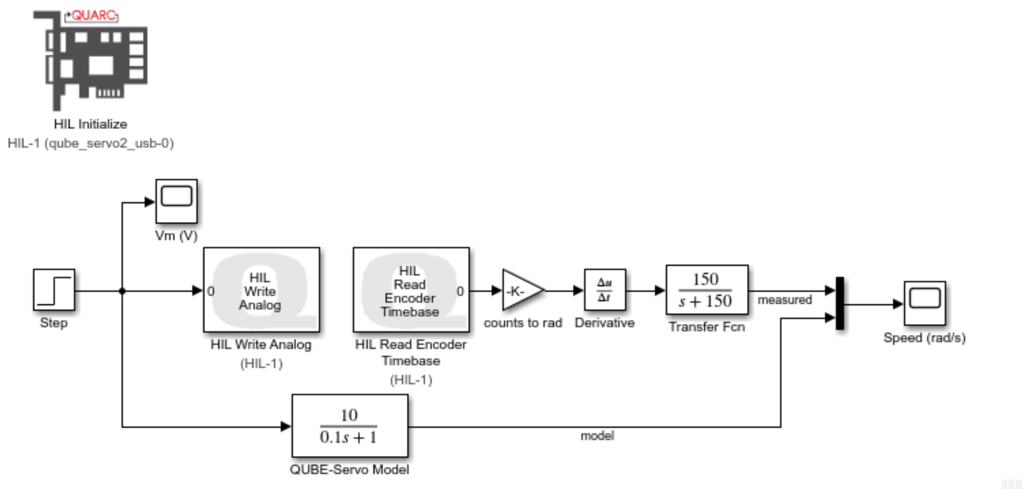


Figure 2.3: Validating step response model

6. Did you derive the model parameters K and τ correctly? Explain.

7. Stop the QUARC® controller.

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