FREQUENCY RESPONSE MODELING

Topics Covered

- Steady-state analysis.
- Magnitude response analysis.
- Phase delay analysis.

Prerequisites

- Hardware Interfacing laboratory experiment.
- Filtering laboratory experiment.

1 Background

An input sinusoid $V_m(t)$ can be characterized by its amplitude and frequency. When applying an input sine wave to a DC motor, the resulting output of the DC motor will be *scaled* and *delayed* sinusoid of the same frequency. For example, in Figure 1.1, t_1 denotes the length of a period of the sinusoid and t_2 shows the time delay between an input voltage signal, V_m, and a *scaled* output speed signal, Ω_m.

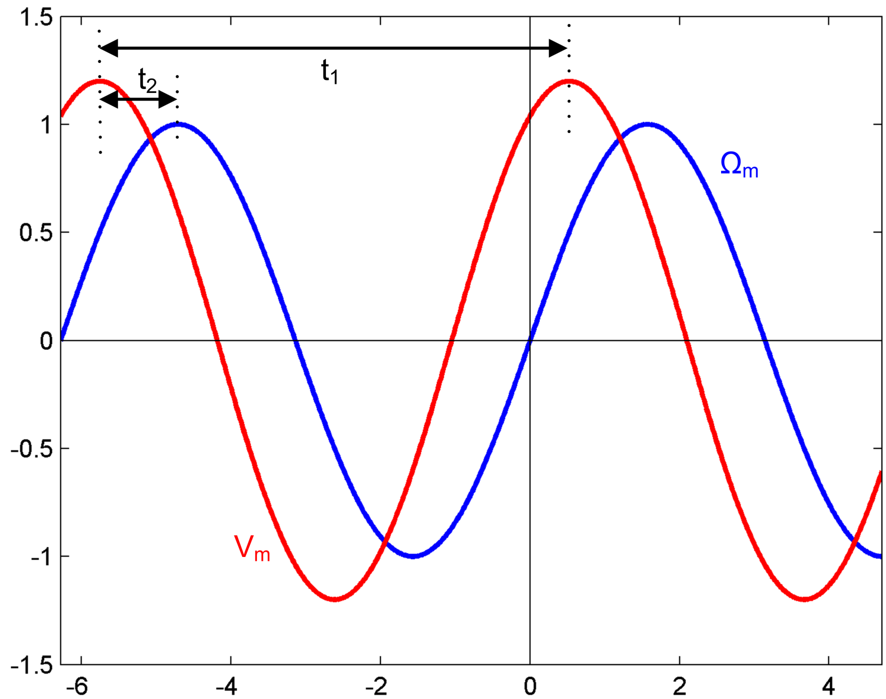


Figure 1.1: Period and phase delay of sinusoidal signals

1.1 Magnitude response analysis

The magnitude response of the resulting output changes with respect to the frequency of the applied sinusoid and can be used to find the time constant τ of the DC motor with transfer function as shown in Equation 1.1. In particular, a Bode plot of the system's magnitude response similar to Figure 1.2 can be obtained by varying the frequency of the applied sinusoid.

One of the characteristics of a Bode plot is the cutoff frequency ω_c, which is defined as the frequency where the gain of the system is 3 dB less than the maximum steady-state gain. An attenuation of 3 dB corresponds to the half-power point of the system, where the power at the cutoff frequency (in dB) is defined as $10\log_{10}(0.5) \approx -3.01$ dB. The corresponding magnitude at the cutoff frequency is then $1/\sqrt{2} \approx 0.707$, or $20\log_{10}(\sqrt{2}/2) \approx -3.01$ dB, of the maximum system gain. The cutoff frequency is also referred to as the *bandwidth* of a system and is a measure for how fast the system responds to a given input.

Recall the first order linear time-invariant (LTI) model of the input voltage-to-speed DC motor transfer function:

$$P(s) = \frac{\Omega_m(s)}{V_m(s)} = \frac{K}{\tau s + 1} \tag{1.1}$$

where K is the steady-state gain, τ is the time constant of the DC motor, and $\Omega_m(s) = \mathcal{L}[w_m(t)]$ and $V_m(s) = \mathcal{L}[v_m(t)]$ are the Laplace transforms of the motor disk speed and the motor input voltage, respectively. The time

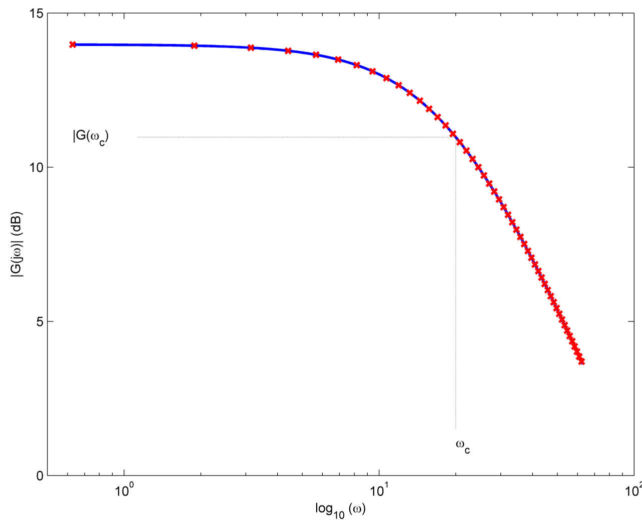


Figure 1.2: Magnitude Bode plot

constant τ can be obtained by applying sinusoidal inputs and observing the system response.

Substituting $s = j\omega$ in Equation 1.1, the magnitude of the frequency response at a frequency ω of the motor input voltage is then defined as

$$|G(\omega)| = \left|\frac{\Omega_m(j\omega)}{V_m(j\omega)}\right| = \left|\frac{K}{\tau j\omega + 1}\right|. \tag{1.2}$$

The magnitude or absolute value of a complex number $z = x + jy$ is defined as

$$|z| = |x + jy| = \sqrt{\Re z^2 + \Im z^2} = \sqrt{x^2 + y^2}. \tag{1.3}$$

Therefore, determining the absolute value of the right side of Equation 1.2 subsequently leads to the following expression for the system's magnitude response with respect to the input frequency:

$$|G(\omega)| = \frac{K}{\sqrt{1 + \tau^2\omega^2}}. \tag{1.4}$$

The system's steady-state (or low frequency) gain can then be obtained by setting $\omega = 0$, i.e. applying a constant signal:

$$K = |G(0)|. \tag{1.5}$$

1.2 Phase delay analysis

Another way to determine τ is to perform a phase delay analysis, i.e. to investigate by how much the system's response lags the system's sinusoidal input. The phase delay (or phase angle) of a complex number $z = x + jy$ is defined as

$$\angle z = \tan^{-1}\frac{\Im\{z\}}{\Re\{z\}} = \tan^{-1}\frac{y}{x}. \tag{1.6}$$

The phase angle, or phase delay, of a transfer function is

$$\Phi_d = \Phi_{num} - \Phi_{den}. \tag{1.7}$$

where Φ_{num} and Φ_{den} represent the phase angle/delay of the numerator and denominator of the transfer function, respectively.

The effect of this phase delay can also be observed in the delay of input/output graphs of Figure 1.1. Here, the phase shift can be expressed as

$$\Phi_d = -\frac{t_2}{t_1} \times 360^\circ. \tag{1.8}$$

2 In-Lab Exercise

In the first part of this experiment, you will determine the system gain K by applying a steady state input. In the second part, you will use sinusoidal inputs to gain information about the magnitude response of your system so that you can draw a Bode magnitude plot to determine the time constant τ. As an alternative way to determine the time constant τ, you will perform a phase shift analysis of your system in the last part of this exercise.

The q_qube2_freq_rsp Simulink model shown in Figure 2.1 applies a sine wave and/or constant voltage to the motor and measures the corresponding speed on the QUBE-Servo 2.

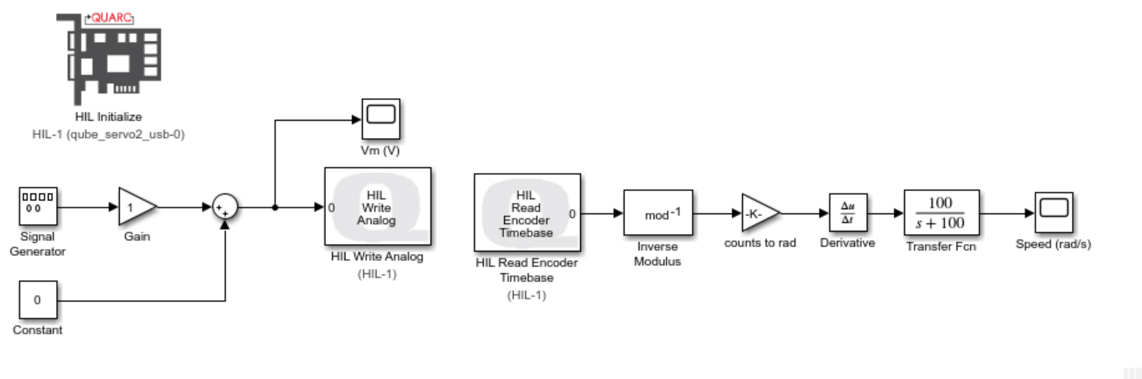


Figure 2.1: Simulink model used to obtain the frequency response of the QUBE-Servo 2

2.1 Steady-state gain

As discussed in the background section, the steady state gain K of the system can be observed by applying an input signal with a frequency of $\omega = 0$ rad, see Equation 1.5.

1. Open q_qube2_freq_rsp Simulink model or design your own based on the model constructed in Filtering laboratory experiment.
2. To apply a constant 3V voltage command, set the Constant block to a value of 3 and the Gain block to 0 (i.e. no sine wave).
3. To apply a constant voltage command to the QUBE-Servo 2, set the Offset block to a value of 3. Build and run the QUARC Controller. The response should look similar to Figure 2.2. Attach the voltage and speed response you obtained.
4. Measure the speed of the load disk and calculate the steady-state gain of the system, K, in rad/s/V (linear and decibel dB).
5. Stop the QUARC controller.

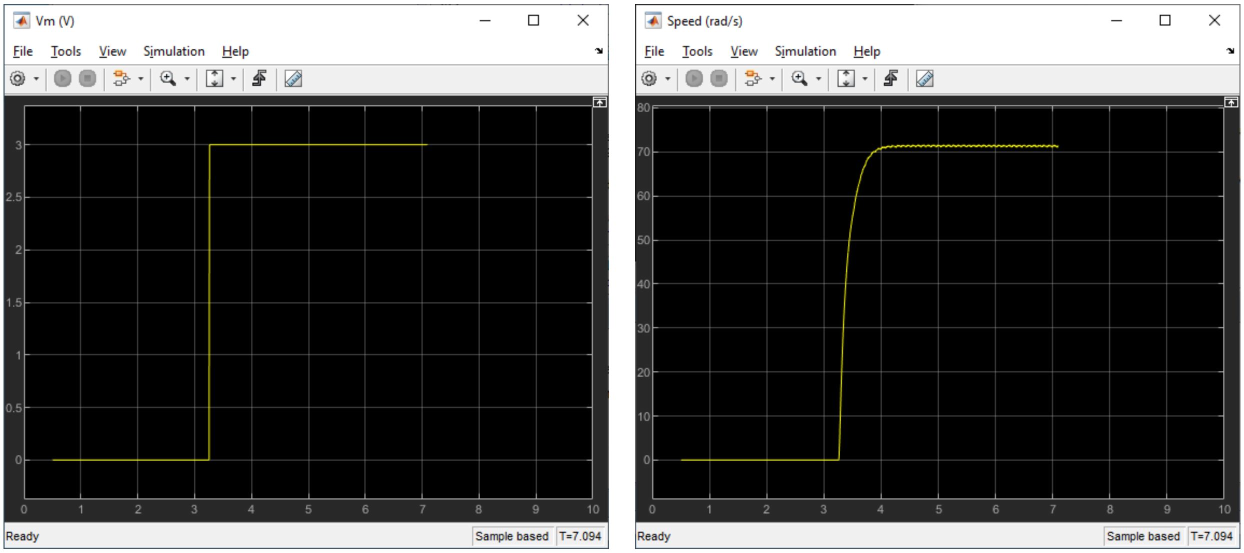
2.2 Magnitude response analysis

In this part of the lab, you will use sinusoidal inputs with different frequencies to determine the time constant, τ, of the system.

1. Given Equation 1.4, derive an expression to determine the time constant, τ.

   Hint: Begin by evaluating the magnitude of the transfer function at the cutoff frequency, ω_c.

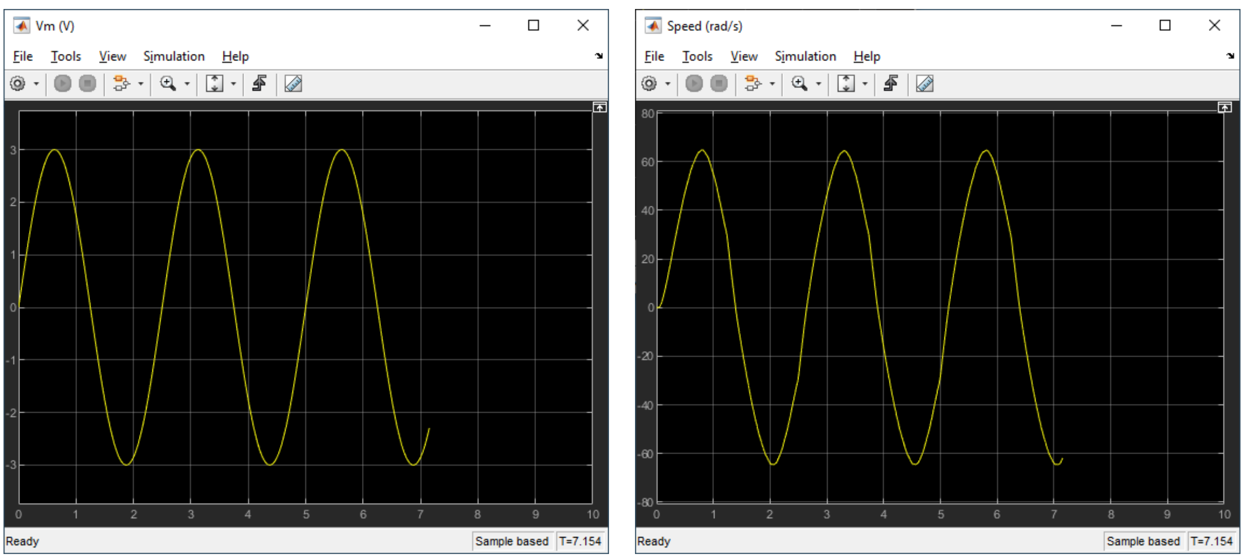
2. To configure the q_qube2_freq_rsp model to apply a sine wave, set the Constant block to 0 and the Gain block to 3. Set the *Frequency* in the Signal Generator block to 0.4 Hz.



(a) Constant input voltage (b) Load disk speed response

Figure 2.2: System response when applying a constant 3V input voltage

3. Run the QUARC controller. An example response is shown in Figure 2.3.



(a) Constant input voltage (b) Load disk speed response

Figure 2.3: System response when applying a 3V 0.4 Hz sine wave voltage

4. Measure the maximum positive speed of the response and compute the gain of the system (linear and in dB). Enter the result in Table 2.1 below.
5. Repeat the previous steps for the remaining frequencies in Table 2.1. Enter the results from the steady-state analysis in the previous section in the line where the frequency is 0.0 Hz.
6. Using the `plot` command and the data you have collected in the previous steps and summarized in Table 2.1, generate a Bode magnitude plot. The amplitude scale should be in decibels (dB) and the frequency scale should be logarithmic.

   Note: Ignore the entry for a frequency of 0 Hz when drawing the Bode plot. The logarithm of 0 is not defined.

f (Hz)	Max Amplitude (V)	Max Load Speed (rad/s)	Gain:|G(ω)| (rad/s/V)	Gain:|G(ω)| (dB)
0.0				
0.4				
0.8				
1.2				
1.6				
2.0				
2.4				
2.8				

Table 2.1: Collected frequency response data.

7. Calculate the time constant, τ, using the obtained Bode magnitude plot. Label the location of the -3 dB cutoff frequency.

   Hint: Use the MATLAB Figure *Data Tips* tool to obtain the values directly from the plot.

2.3 Phase Delay Analysis

Phase delay analysis will be used in this experiment to determine the time constant, τ, of the QUBE-Servo 2. The q_qube2_phase_delay Simulink model shown in Figure 2.4 applies a sinusoidal voltage to the motor and measures the corresponding speed on the QUBE-Servo 2. The input and output signal are plotted together in the Time Delay scope to measure the time delay.

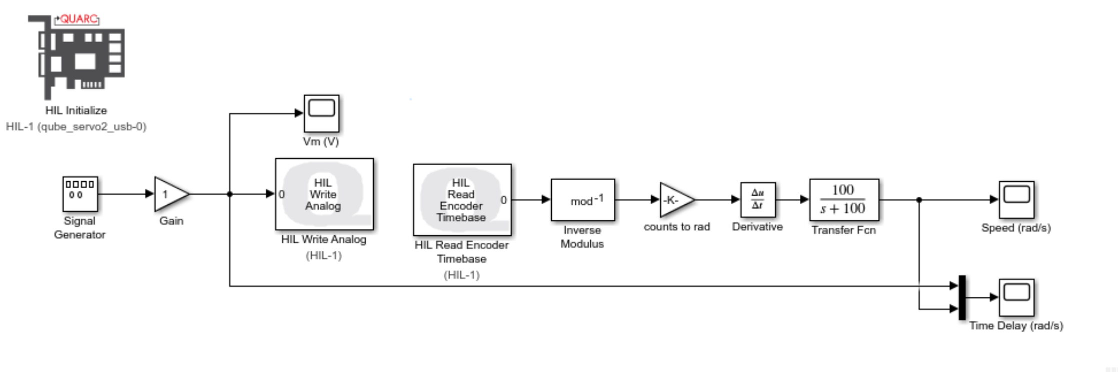
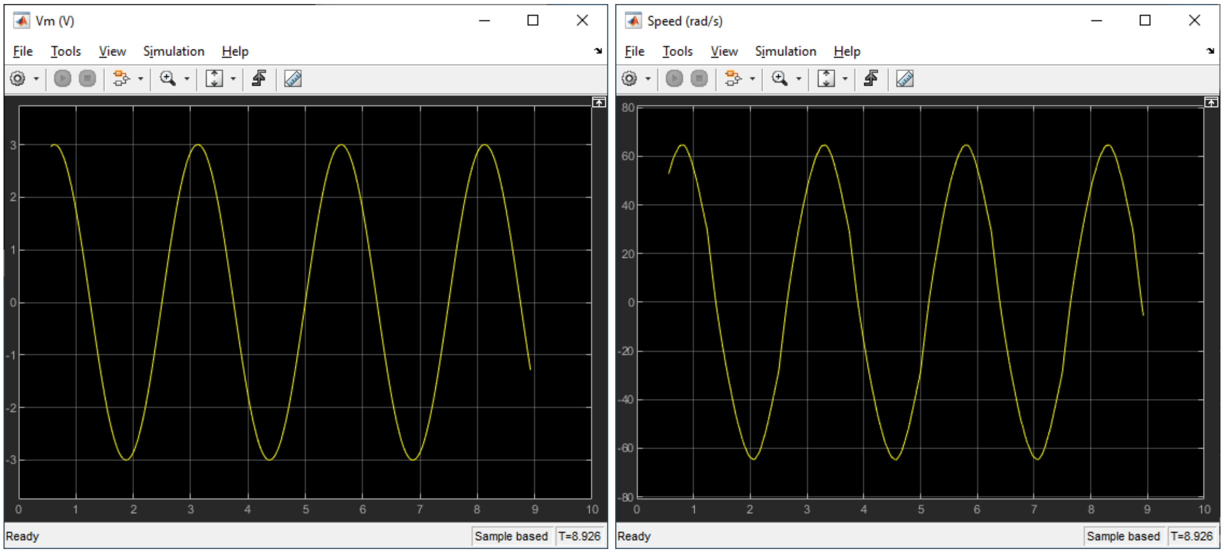


Figure 2.4: Simulink model used to measure the phase delay of the QUBE-Servo 2

1. For the input voltage-to-speed transfer function as in Equation 1.1, find an expression for the time constant, τ, in terms of the frequency of the input sinusoid and the resulting phase delay.
2. Express the time constant equation found in Step 1 directly with the time delay of the input and output signals.
3. Open the q_qube2_phase_delay Simulink model.
4. Configure the model to apply a 3 V 0.4 Hz sine wave to the motor by setting the *Frequency* in the Signal Generator to 0.4 Hz and the Gain block to 3.
5. Build and run the QUARC controller. A sample response is shown in Figure 2.5



(a) Input voltage

(b) Measured speed

Figure 2.5: Response when appling a 3 V sine wave at 0.4 Hz

6. Measure the time delay of the speed output when applying a 3 V at 0.4 Hz sinusoidal input. Attach the response of the $\mathrm{Time\ Delay}$ scope you obtained.
7. Determine the corresponding phase shift in degrees and radians. Based on these measurements, compute the time constant, τ, for the QUBE-Servo 2.
8. Compare the time constant found using phase delay analysis with the result obtained previously using the Bode plot. If they are different, list one source that may have contributed to the different results.

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