Stability Analysis

Topics Covered

- Stable, marginally stable, and unstable systems.
- Open-loop position and speed response of a servo.

Prerequisites

- Integration laboratory experiment.
- Filtering laboratory experiment.

1 Background

1.1 Servo Model

The QUBE-Servo 2 voltage-to-speed transfer function is

$$P_{v-s}(s) = \frac{\Omega_m(s)}{V_m(s)} = \frac{K}{\tau s + 1}, \tag{1.1}$$

where $K = 23.8$ rad/(V-s) is the model steady-state gain, $\tau = 0.13$ s is the model time constant, $\Omega_m(s) = \mathcal{L}[\omega_m(t)]$ is the motor speed (i.e. speed of load disk), and $V_m(s) = \mathcal{L}[v_m(t)]$ is the applied motor voltage. If desired, you can conduct an experiment to find more precise model parameters, K and τ, for your particular servo (e.g. performing the Bump Test Modeling lab).

The voltage-to-position process transfer function the same as Equation 1.1 with an integrator in series

$$P_{v-p} = \frac{\Theta_m(s)}{V_m(s)} = \frac{K}{s(\tau s + 1)} \tag{1.2}$$

where $\Theta_m(s) = \mathcal{L}[\theta_m(t)]$ is the load gear position.

1.2 Stability

Definition for Bounded-Input Bounded-Output (BIBO) stability is:

1. A system is stable if every bounded input yields a bounded output.
2. A system is unstable if any bounded input yields a unbounded output.

The stability of a system can be determined from its poles:

- Stable systems have poles only in the left-hand plane.
- Unstable systems have at least one pole in the right-hand plane and/or poles of multiplicity greater than 1 on the imaginary axis.
- Marginally stable systems have one pole on the imaginary axis and the other poles in the left-hand plane.

2 In-Lab Exercises

Based on the models already designed in the QUBE-Servo 2 Integration and Filtering laboratory experiment, design a model that applies a step of 1 V to the motor and reads the servo velocity and the position as shown in Figure 2.1.

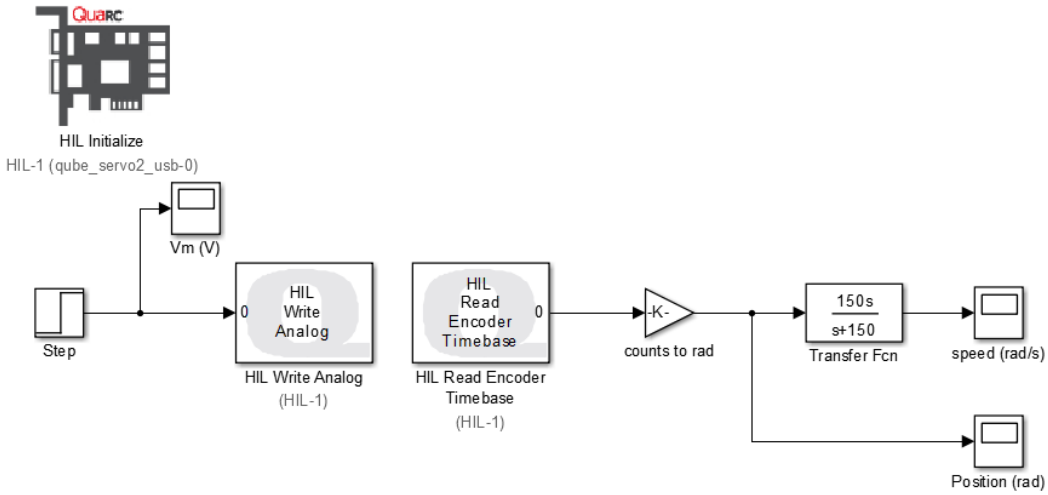
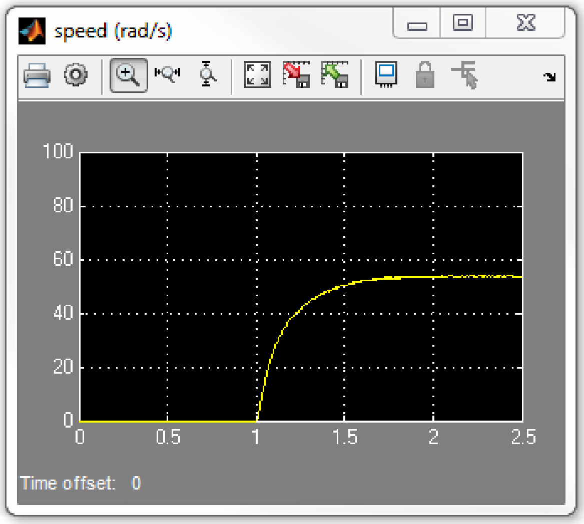
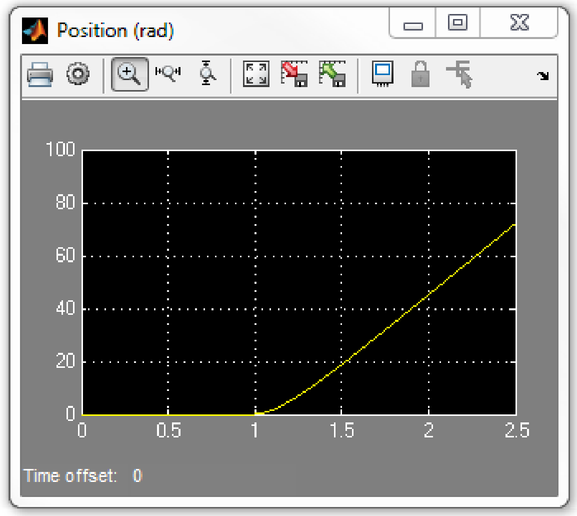


Figure 2.1: Measuring speed and position when applying a step

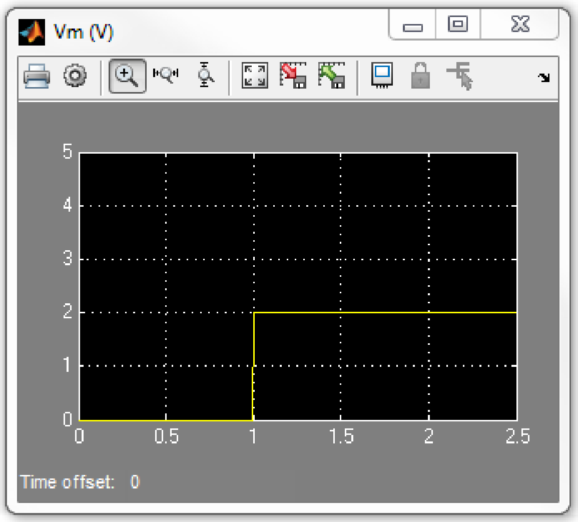
1. Determine the stability of the voltage-to-speed servo system from its poles.
2. Determine the stability of the voltage-to-position servo system from its poles.
3. Apply a unit step voltage to the servo by running the QUARC® model shown in Figure 2.1. The position and speed step response should be similar to Figure 2.2.



(a) Speed Response



(b) Position Response



(c) Voltage Input

Figure 2.2: Step Response

4. Based on the *speed* response and the BIBO principle, what is the stability of the system? How does this compare with your results from the pole analysis. Similarly, assess the stability of the system using the *position* response using BIBO and the pole analysis.
5. Based on the *position* response and the BIBO principle, what is the stability of the system? How does this compare with your results from the pole analysis.
6. Is there an input where the open-loop servo position response is stable? If so then modify the Simulink diagram to include your input, test it on the servo, and show the position response. Based on this result, how could you define marginal stability in terms of bounded inputs?

   Hint: Try an impulse (i.e. short step) or sinusoid input and compare the position response with the step response observed earlier.
7. Stop the QUARC® controller.

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