SECOND-ORDER SYSTEMS

Topics Covered

- Underdamped second-order systems.
- Damping ratio and natural frequency.
- Peak time and percent overshoot time-domain specifications.

Prerequisites

- Hardware Interfacing laboratory experiment.
- Filtering laboratory experiment.

1 Background

1.1 Second-Order Step Response

The *standard second-order* transfer function has the form

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \tag{1.1}$$

where ω_n is the natural frequency and ζ is the damping ratio. The properties of its response depend on the values of the parameters ω_n and ζ.

Consider a second-order system as shown in Equation 1.1 subjected to a step input given by

$$R(s) = \frac{R_0}{s},$$

with a step amplitude of $R_0 = 1.5$. The system response to this input is shown in Figure 1.1, where the red trace is the output response $y(t)$ and the blue trace is the step input $r(t)$.

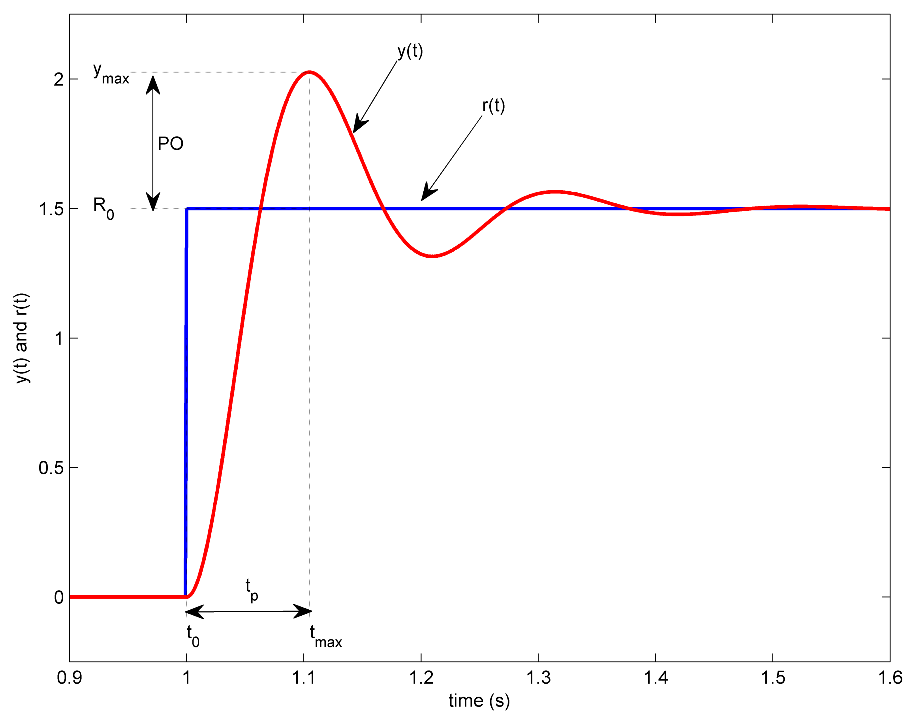


Figure 1.1: Standard second-order step response

1.2 Peak Time and Overshoot

The maximum value of the response is denoted by the variable y_{max} and it occurs at a time t_{max}. For a response similar to Figure 1.1, the percent overshoot is found using

$$PO = \frac{100\,(y_{max} - R_0)}{R_0}. \tag{1.2}$$

From the initial step time, t_0, the time it takes for the response to reach its maximum value is

$$t_p = t_{max} - t_0. \tag{1.3}$$

This is called the *peak time* of the system.

In a second-order system, the amount of overshoot depends solely on the damping ratio parameter and it can be calculated using the equation

$$PO = 100e^{\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)}. \tag{1.4}$$

The peak time depends on both the damping ratio and natural frequency of the system and it can be derived as:

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}. \tag{1.5}$$

Generally speaking, the damping ratio affects the shape of the response while the natural frequency affects the speed of the response.

1.3 Unity Feedback

The unity-feedback control loop shown in Figure 1.2 will be used to control the position of the QUBE-Servo 2.

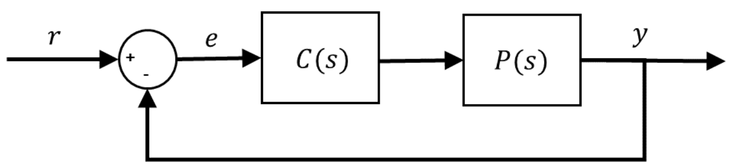


Figure 1.2: Unity feedback loop

The QUBE-Servo 2 voltage-to-position transfer function is

$$P(s) = \frac{\Theta_m(s)}{V_m(s)} = \frac{K}{s(\tau s + 1)}. \tag{1.6}$$

where K is the model steady-state gain, τ is the model time constant, $\Theta_m(s) = \mathcal{L}\left[\theta_m(t)\right]$ is the motor / disk position, and $V_m(s) = \mathcal{L}\left[v_m(t)\right]$ is the applied motor voltage. The default values for the model parameters are $K = 22.4$ rad/s/V and $\tau = 0.15$ s.

The controller is denoted by $C(s)$. In this lab, we are only going to use unity feedback therefore

$$C(s) = 1. \tag{1.7}$$

The closed-loop transfer function of the QUBE-Servo 2 position control from the reference input $R(s) = \Theta_d(s)$ to the output $Y(s) = \Theta_m$ using unity feedback as shown in Figure 1.2 is

$$\frac{\Theta_d(s)}{V_m(s)} = \frac{\frac{K}{\tau}}{s^2 + \frac{1}{\tau}s + \frac{K}{\tau}}. \tag{1.8}$$

2 In-Lab Exercises

Design a SIMULINK® model similar to Figure 2.1. This implements the unity feedback control given in Figure 1.2 in SIMULINK®. A step reference (i.e. desired position or setpoint) of 1 rad is applied at 1 second and the controller runs for 2.5 seconds.

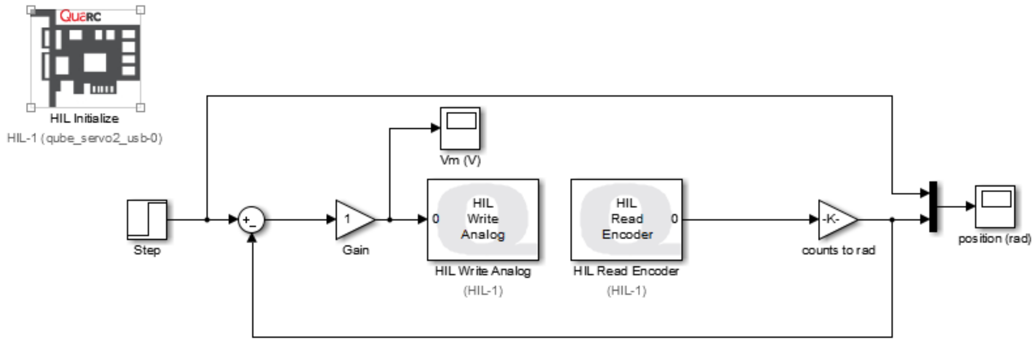
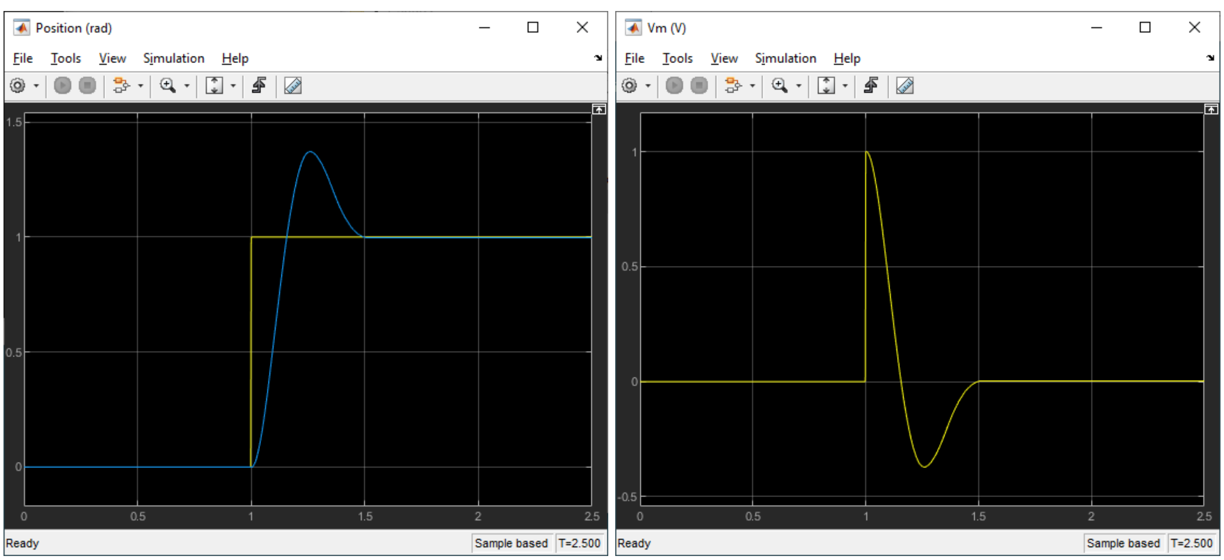


Figure 2.1: Unity feedback position control of QUBE-Servo 2

1. Given the QUBE-Servo 2 closed-loop equation under unity feedback in Equation 1.8 and the default model parameters given above or, preferably, using more precise model parameters found by going through one of the modeling experiments (e.g. Step Response Modeling laboratory experiment), find the natural frequency and damping ratio of the system.
2. Based on your obtained ω_n and ζ, what is the expected peak time and percent overshoot?
3. Build and run the QUARC® controller. The scopes should look similar to Figure 2.2.



(a) Position (b) Voltage

Figure 2.2: Unity feedback QUBE-Servo 2 step response

4. Attach the QUBE-Servo 2 position response - showing both the setpoint and measured positions in one scopes - as well as the motor voltage.

   Hint: For information on saving data to MATLAB® for offline analysis, see the QUARC® help documentation (under *QUARC Targets* | *User's Guide* | *QUARC Basics* | *Data Collection*). You can then use the MATLAB® `plot` command to generate the necessary MATLAB figure.

5. Measure the peak time and percent overshoot of the response and compare that with expected results.

   Hint: Use the *Cursor Measurements* tool in the SIMULINK® Scope to take measure points off the scope directly.

6. Name one possible reason why the *expected* peak time and percent overshoot calculated do not match the measured values exactly?

7. Make sure the QUARC® controller is stopped.

8. Power off the QUBE-Servo 2 if no more experiments will be ran in this session.

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