

# ROTARY PENDULUM MODELING

## Topics Covered

- Interact with the **QUANSER**<sup>®</sup> QUBE-Servo 2 Rotary Pendulum system.
- Configure sensor and actuator gains to match model conventions.

## Prerequisites

- Hardware Interfacing laboratory experiment.

# 1 Background

The rotary pendulum system, also known as the Furuta Pendulum, is a classic system often used to teach modeling and control in physics and engineering. The convention used for the modeling of the QUBE-Servo 2 rotary pendulum is shown in Figure 1.1.

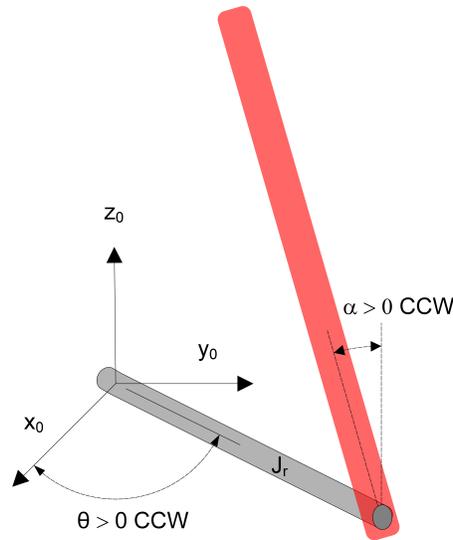


Figure 1.1: Model conventions of rotary pendulum

The rotary arm, which is attached to the motor pivot, is denoted by the variable  $\theta$  and the pendulum angle, attached to the end of the rotary arm, is denoted by  $\alpha$ . Note the following conventions:

- Angle  $\alpha$  is defined as the *inverted pendulum angle*, i.e. the angle with respect to the upright vertical position where  $\alpha = 0$  means the pendulum is perfectly upright. It is expressed mathematically using

$$\alpha = \alpha_{full} \bmod 2\pi - \pi. \quad (1.1)$$

where  $\alpha_{full}$  is the pendulum angle measured by the encoder, i.e. the continuous angle measurement defined as zero when pendulum is in the downward configuration.

- Both angles are defined as positive when rotated in the counter-clockwise (CCW) direction.
- When a positive voltage is applied to the motor, the rotary arm moves in the positive CCW direction.

The goal is to design a system model that follows these conventions. The Hardware Interfacing laboratory experiment introduced the DC motor and encoders on the QUBE-Servo 2 system. The pendulum angle is also measured using an encoder.

## 2 In-Lab Exercises

In this lab, we will make a **SIMULINK®** model using **QUARC®** blocks to drive to the dc motor and measure both the rotary arm and pendulum angles - similarly as shown in Figure 2.1.

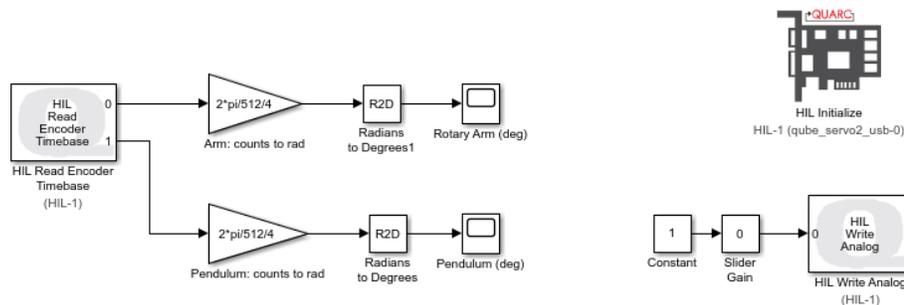
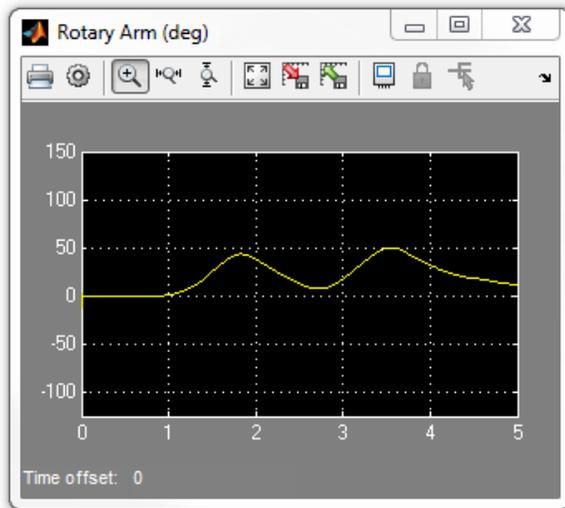
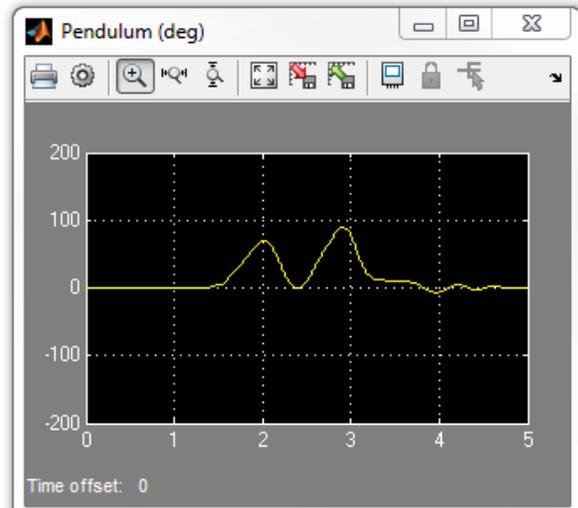


Figure 2.1: **SIMULINK®** model used with **QUARC®** to drive motor and read angles on QUBE-Servo 2 Rotary Pendulum system

1. Using the **SIMULINK®** model you made in the Hardware Interfacing laboratory experiment, do the following:
  - Configure the HIL Read Encoder Timebase block to read from both channels #0 and #1. The pendulum is measured on channel #1.
  - Setup the encoder gains on each channel to read the angles in radians (instead of degrees as in the lab).
  - Connect the the measured angles to scopes, but display them in degrees. You can do this using the *Simulink Extras | Transformation* blocks or just by adding another Gain block that converts radians to degrees.
  - If desired, add a Slider Gain block between the Constant and Analog Output block. Make sure the Constant source block is set to 1 in this case.
2. Build and run the **QUARC®** controller.
3. Rotate the rotary arm and pendulum counter-clockwise and examine the response on the scopes. Example responses are shown in Figure 2.2. Do the measured angles follow the modeling conventions given in Section 1



(a) Rotary Arm



(b) Pendulum

Figure 2.2: Measured rotary arm and pendulum angles

4. Apply a small voltage (e.g. 0.5 V) to the motor by changing the Constant block connected to Analog Output. Does this adhere to the modeling conventions?
5. Modify the Simulink model such that the measured angles and applied voltage follow by the modeling conventions. Briefly list any changes made.
6. Add modulus and bias blocks, as shown in Figure 2.3, to measure *inverted pendulum angle*, defined as Equation 1.1.

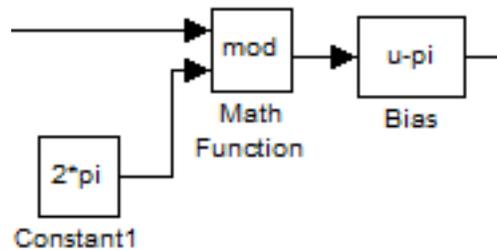


Figure 2.3: SIMULINK® modulus and bias blocks

7. Make sure the pendulum is hanging in the downward position and lies motionless before starting the controller.
8. Build and run the QUARC® controller.
9. Rotate the pendulum to the upright vertical position and ensure the angle is measured correctly and it follows the model conventions shown in Figure 1.1. Capture the response of the pendulum being raised to the inverted position. Explain what the bias and modulus functions do?
10. Stop the QUARC® controller.

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