PD CONTROL

Topics Covered

- Servo position control.
- Proportional-derivative (PD) compensator.
- Designing control according to specifications.

Prerequisites

- Hardware Interfacing laboratory experiment.
- Filtering laboratory experiment.
- Stability Analysis laboratory experiment.

1 Background

1.1 Servo Model

The QUBE-Servo 2 voltage-to-position transfer function is

$$P(s) = \frac{\Theta_m(s)}{V_m(s)} = \frac{K}{s(\tau s + 1)}, \tag{1.1}$$

where $K = 22.4$ rad/(V-s) is the model steady-state gain, $\tau = 0.15$ s is the model time constant, $\Theta_m(s) = \mathcal{L}[\theta_m(t)]$ is the motor / disk position, and $V_m(s) = \mathcal{L}[v_m(t)]$ is the applied motor voltage. If desired, you can conduct an experiment to find more precise model parameters, K and τ, for your particular servo (e.g. performing the Step Response Modeling laboratory experiment).

1.2 PID Control

The proportional, integral, and derivative control can be expressed mathematically as follows

$$u(t) = k_p e(t) + k_i \int_0^t e(\tau) d\tau + k_d \frac{de(t)}{dt}. \tag{1.2}$$

The corresponding block diagram is given in Figure 1.1. The control action is a sum of three terms referred to as proportional (P), integral (I) and derivative (D) control gain. The controller Equation 1.2 can also be described by the transfer function

$$C(s) = k_p + \frac{k_i}{s} + k_d s. \tag{1.3}$$

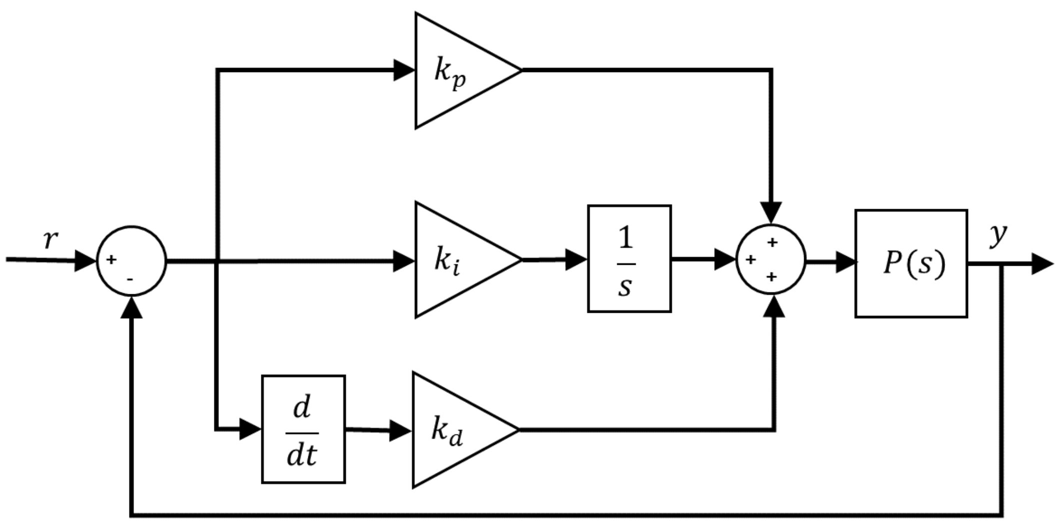


Figure 1.1: Block diagram of PID control

The functionality of the PID controller can be summarized as follows. The proportional term is based on the present error, the integral term depends on past errors, and the derivative term is a prediction of future errors.

The PID controller described by Equation 1.2 or Equation 1.3 is an *ideal* PID controller. However, attempts to implement such a controller may not lead to a good system response for real-world system. The main reason for this is that measured signals often includes measurement noise. Differentiating a noisy measured signal (i.e. in the derivative control) can produce a control signal with high-frequency components that yields an undesirable response and, over time, may even damage the actuator (e.g. DC motor).

1.3 PD Position Control

For this laboratory, the integral term will not be used to control the servo position. A variation of the classic PD control will be used: the proportional plus rate feedback control illustrated in Figure 1.2, also known as proportional plus velocity or PV control.

Unlike the standard PD, only the negative velocity is fed back (i.e. not the velocity of the *error*). Further, in the hardware implementation of the control a low-pass filter will be used in-line with the derivative term to suppress measurement noise.

The combination of a first order low-pass filter, $\omega_f/(s+\omega_f)$, and the derivative term, s, will be used instead of a direct derivative:

$$D(s) = \frac{\omega_f s}{s+\omega_f} \tag{1.4}$$

where ω_f is the cut-off frequency of the filter.

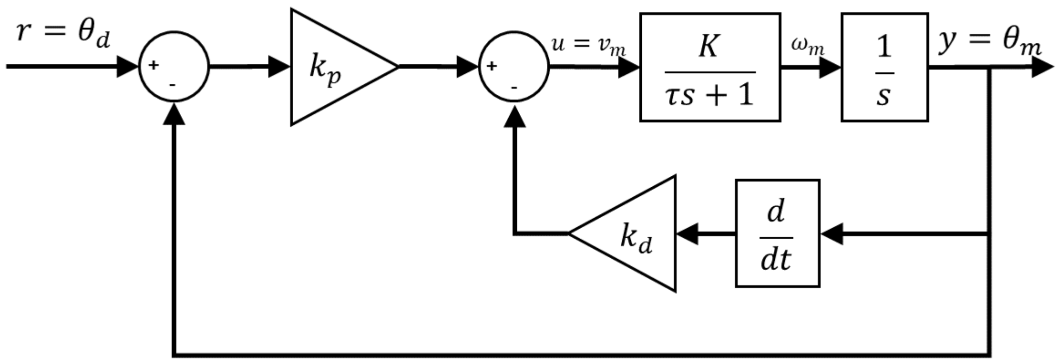


Figure 1.2: Block diagram of proportional plus rate feedback control

The proportional plus rate feedback control has the following structure

$$u = k_p\left(r(t) - y(t)\right) - k_d\dot{y}(t), \tag{1.5}$$

where k_p is the proportional gain, k_d is the derivative (velocity) gain, $r = \theta_d(t)$ is the setpoint or reference motor / load angle, $y = \theta_m(t)$ is the measured load shaft angle, and $u = V_m(t)$ is the control input (applied motor voltage).

The closed-loop transfer function of the QUBE-Servo 2 is denoted $Y(s)/R(s) = \Theta_m(s)/\Theta_d(s)$. Assume all initial conditions are zero, i.e. $\theta_m(0^-) = 0$ and $\dot{\theta}_m(0^-) = 0$, taking the Laplace transform of Equation 1.5 yields

$$U(s) = k_p(R(s) - Y(s)) - k_d sY(s),$$

which can be substituted into Equation 1.1 to result in

$$Y(s) = \frac{K}{s\left(\tau s + 1\right)}\left(k_p\left(R(s) - Y(s)\right) - k_d sY(s)\right).$$

Solving for $Y(s)/R(s)$, we obtain the closed-loop expression

$$\frac{Y(s)}{R(s)} = \frac{Kk_p}{\tau s^2 + (1 + Kk_d)\,s + Kk_p}. \tag{1.6}$$

This is a second-order transfer function. Recall the standard second-order transfer function

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}. \tag{1.7}$$

2 In-Lab Exercises

The Simulink model shown in Figure 2.1 implements the proportional plus rate feedback, or PV, control outlined in 1.3. As discussed, the control includes a filter with the derivative, $100s/(s+100)$, instead of using a direct derivative.

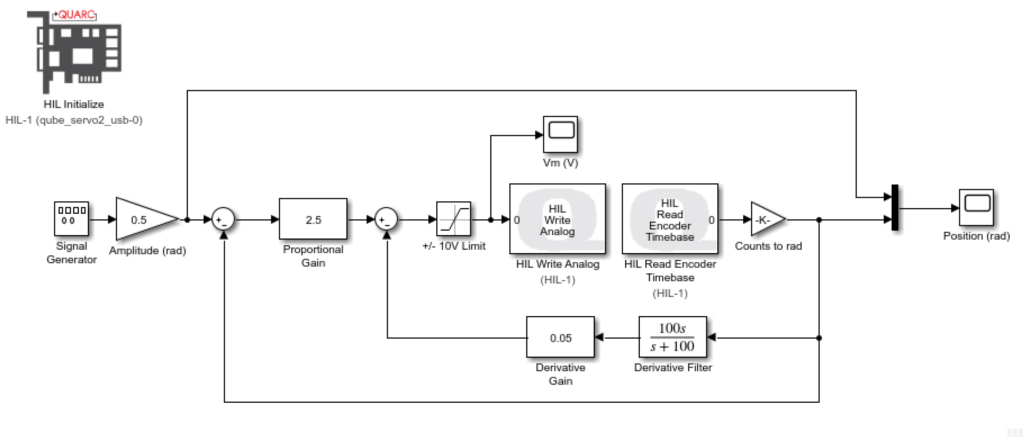
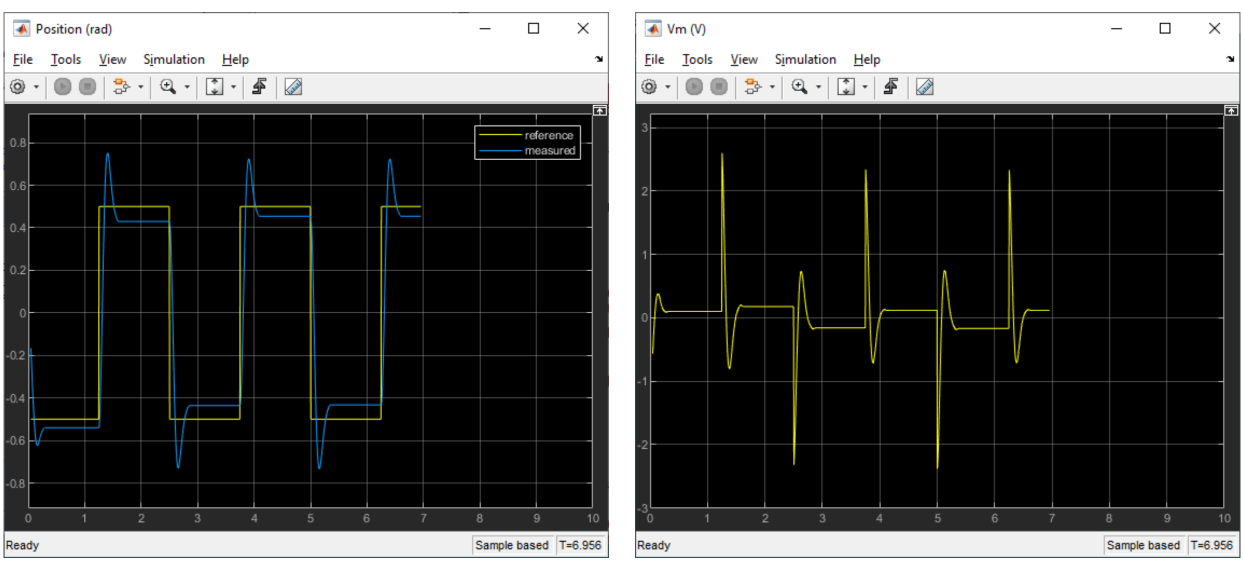


Figure 2.1: Implementation of proportional and rate feedback control on QUBE-Servo 2

1. Open the Simulink model shown in Figure 2.1. Alternatively, design your own using the Filtering laboratory experiment as a basis.
2. Set the Signal Generator block such that the servo command (i.e. reference angle) is a square wave with an amplitude of 0.5 rad and at a frequency of 0.4 Hz.
3. Set the proportional gain to $k_p = 2.5$ V/rad and the derivative gain to $kd = 0.05$ V-s/rad.
4. Build and run the QUARC® controller. The response should look similarly as shown in Figure 2.2.



(a) Position

(b) Voltage

Figure 2.2: QUBE-Servo 2 PD control with $k_p = 2.5$ V/rad and $k_d = 0.05$ V/(rad/s).

5. Set $k_p = 2.5$ V/rad and $k_d = 0$ V/(rad/s). Keep the derivative gain at 0 and vary k_p between 1 and 4. How does the proportional gain affect the servo position control response?

6. Set $k_p = 2.5$ V/rad and vary the derivative gain k_d between 0 and 0.15 V$/($rad$/$s$)$. How does the derivative gain affect the servo position control response?
7. Stop the QUARC® controller.
8. Find the proportional and derivative gains required for the QUBE-Servo 2closed-loop transfer function given in Equation 1.6 to match the standard second-order system in Equation 1.7. Your gain equations will be a function of ω_n and ζ.
9. For the response to have a peak time of 0.15 s and a percentage overshoot of 2.5 %, the natural frequency and damping ratio needed are $\omega_n = 32.3$ rad/s and $\zeta = 0.76$. Using the QUBE-Servo 2 model parameters K and τ given in Section 1.1 or those you found through one of the modeling labs, e.g. Step Response Modeling laboratory experiment, calculate the control gains needed to satisfy these requirements.
10. Run the controller with the newly designed gains on the QUBE-Servo 2. Attach the position response as well as the motor voltage used.
11. Measure the percent overshoot and peak time of the QUBE-Servo 2 response. Do they match the desired percent overshoot and peak time specifications given in Step 9 without saturating the motor (going beyond ± 10 V)?

    Hint: Use the *Cursor Measurements* tool in the SIMULINK® Scope to take measurements of the response.
12. If your response did not match the above overshoot and peak time specification, try tuning your control gains until your response does satisfy them. Attach the resulting MATLAB® figure, resulting measurements, and comment on how you modified your controller to arrive at those results.
13. Stop the QUARC® controller.

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