ROBUSTNESS AND SENSITIVITY

Topics Covered

- Unity feedback loop.
- Finding the sensitivity transfer function.
- Minimum sensitivity design.

Prerequisites

- Hardware Interfacing laboratory experiment.
- Filtering laboratory experiment.
- Stability Analysis laboratory experiment.
- PD Control laboratory experiment.

1 Background

1.1 Minimum Sensitivity Design

The closed-loop system represented in Figure 1.1 is influenced by two external signals: the reference $R(s)$ and the load disturbance $D(s)$. The compensator or controller is $C(s)$ and the plant transfer function is denoted as $P(s)$.

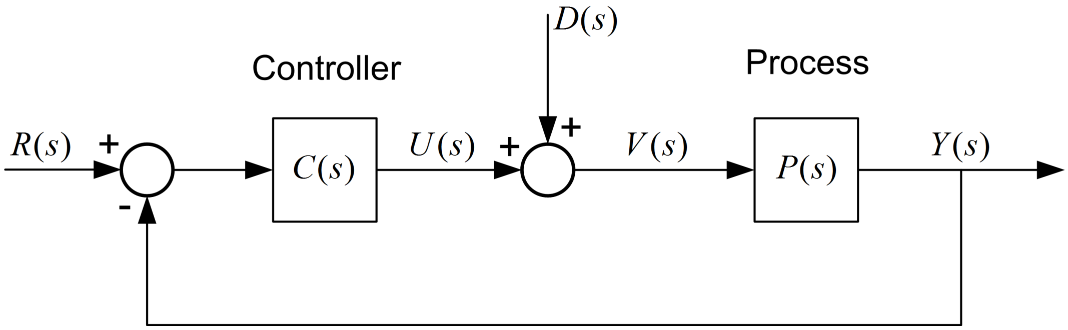

Figure 1.1: Block diagram of a closed-loop control with an added disturbance

The closed-loop transfer function of Figure 1.1 from the reference, r, to the output signal, y, is

$$G_{ry} = \frac{P(s)C(s)}{1 + P(s)C(s)}. \tag{1.1}$$

To investigate how small variations in the process $P(s)$ influence the closed-loop transfer function G_{ry}, we consider the sensitivity function $S(s)$ defined as follows

$$\frac{\Delta G_{ry}(s)}{G_{ry}(s)} = \frac{\Delta P(s)}{P(s)} S(s). \tag{1.2}$$

At the limit, Equation 1.2 becomes

$$S(s) = \frac{\delta G_{ry}}{\delta P} \frac{P(s)}{G_{ry}(s)} \tag{1.3}$$

Differentiating $G_{ry}(s)$ with respect to $P(s)$ results in

$$\frac{dG_{ry}}{dP} = \frac{C}{(1 + PC)^2} \tag{1.4}$$

Substituting the above relation and Equation 1.1 into Equation 1.3 leads to the following expression for the sensitivity function:

$$S = \frac{1}{1 + PC} \tag{1.5}$$

The largest value of the sensitivity function, M_s, is a measure of the sensitivity

$$M_s = \max_{\omega} |S(i\omega)| \tag{1.6}$$

1.2 Sensitivity Transfer Function

The voltage-to-speed transfer function of the QUBE-Servo 2 is given as

$$P(s) = \frac{\Omega_m(s)}{V_m(s)} = \frac{K}{\tau s + 1}, \tag{1.7}$$

where $K = 22.4$ rad/s/V is the model steady-state gain, $\tau = 0.15$ s is the model time constant, $\Omega_m(s) = \mathcal{L}[\omega_m(t)]$ is the angular rate of the load disk, and $V_m(s) = \mathcal{L}[v_m(t)]$ is the applied motor voltage. You can also conduct one of the modeling experiments, e.g. Step Response Modeling laboratory experiment, to find more accurate model parameters, K and τ, for your particular QUBE-Servo 2.

To control the speed of the servo, we will implement a PI control. The block diagram of a generic PI controller in Figure 1.2. The ideal PI controller with proportional gain k_p and an integral gain k_i can be expressed mathematically as

$$u(t) = k_p e(t) + k_i \int_0^t e(\tau) d\tau \tag{1.8}$$

with corresponding transfer function

$$C(s) = \left(k_p + \frac{k_i}{s}\right) E(s). \tag{1.9}$$

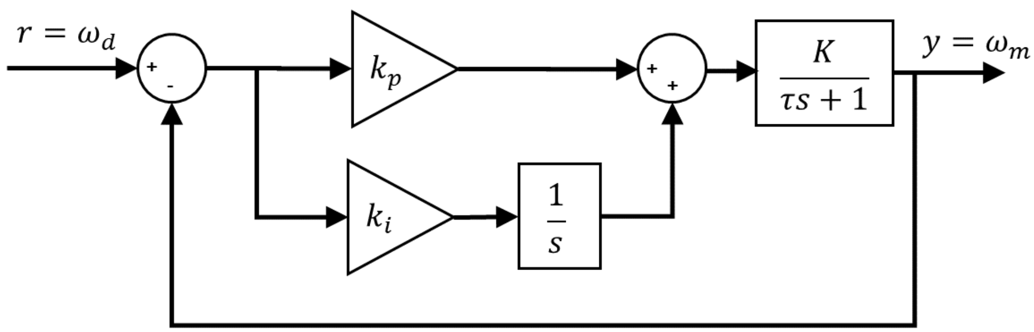

Figure 1.2: Block diagram of PI control for speed control

With the voltage-to-speed transfer function from Equation 1.7 and the PI controller in Equation 1.9, the sensitivity transfer function for the QUBE-Servo 2 becomes

$$S(s) = \frac{1}{1 + \frac{K}{\tau s+1}\left(k_p + \frac{k_i}{s}\right)} = \frac{s(\tau s + 1)}{\tau s^2 + (1 + K k_p)s + K k_i} \tag{1.10}$$

In terms of the standard second-order transfer function discussed in the Stability Analysis laboratory experiment, the sensitivity function can be represented in terms of ω_n and ζ as follows,

$$S(s) = \frac{\left(s + \frac{1}{\tau}\right) s}{s^2 + 2\zeta\omega_n s + \omega_n^2}. \tag{1.11}$$

1.3 Adding Sampling and Filtering Effects

As discussed in the *Filtering Lab*, a low-pass filter is needed to remove the noise from the speed measurement. The first-order low-pass filter transfer function is

$$G_f(s) = \frac{\omega_f}{s + \omega_f} \tag{1.12}$$

where ω_f is the cutoff frequency (in rad/s).

The sampling rate (or loop rate) of the software that implements the PI control is typically ran at 500 Hz. This causes a sample delay of $h = 0.002$ s. Taking both the filter and sampling delay effects into account, the open-loop transfer of the motor becomes

$$P(s) = \frac{K\omega_f e^{-sh}}{(\tau s + 1)(s + \omega_f)} \tag{1.13}$$

Filtering and sampling influence how sensitive and robust the process is to parameter variations and disturbances.

2 In-Lab Exercises

To assess the robustness of the control system, the system will be subjected to a *Loop Gain* and *nDelay* disturbances as illustrated in Figure 2.1. The *Loop Gain* parameter is a pure gain and is used to estimate the system gain margin. The *nDelay* parameter is an integer number of sample periods and is used to determine the extra time delay needed to make the system unstable.

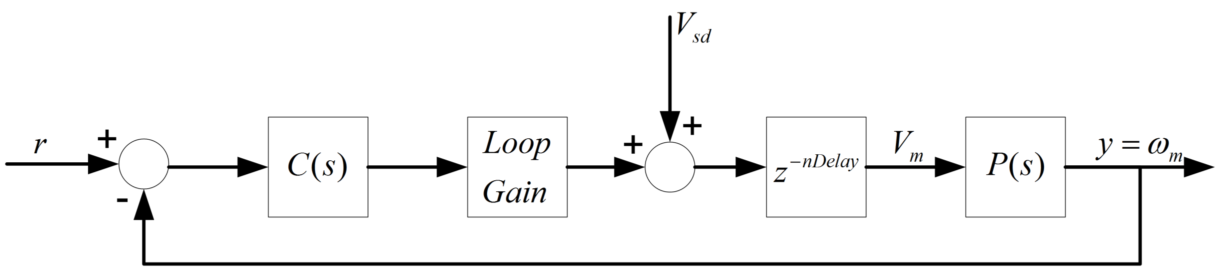

Figure 2.1: PI speed control with *Loop Gain* and *Sample Delay* disturbances

Using the QUARC® controller designed in the Stability Analysis laboratory experiment, build the Simulink model shown in Figure 2.2. This implements the PI control loop shown in Figure 1.2 with the added the *Loop Gain* and *Sample Delay* effects shown in Figure 2.1. The Variable Time Delay block is used to add a time delay to the control signal. The *nDelay* sample delay is converted into time delay by multiplying it by the controller sampling interface (done using the *Sample time (s)* gain block and the `qc_get_step_size` QUARC command).

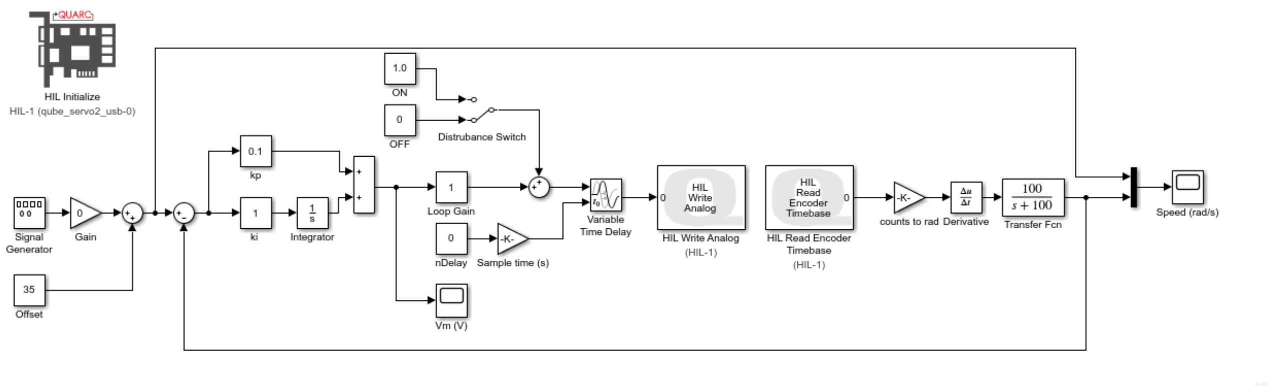

Figure 2.2: Simulink/QUARC model that implements PI speed control with *Loop Gain* and *Sample Delay* effects

Note: The *HIL Read Encoder timebase* block is used for the controller to be triggered by the clock on the data acquisition device. Using hardware-based timing is recommended when doing speed control that derives motor speed computationally from a position sensor (i.e. encoder).

1. Find the proportional and integral gains required for the QUBE-Servo sensitivity transfer function given in Equation 1.11 to match the standard second-order system in Equation 1.11. Your gain equations will be a function of ω_n and ζ.
2. **Speed Lab Design (SLD):** Find the PI gains needed for the $\omega_n = 18.5$ rad/s and $\zeta = 0.8$ specifications. This set of gains are optimized for speed control performance and will be denoted as *Speed Lab Design (SLD)*. Use either the default model parameters given in Section 1 or, for more accurate results, derive the model parameters for the QUBE-Servo 2 by going through one of the modeling laboratories, e.g. Step Response Modeling laboratory experiment.
3. Run the model shown in Figure 2.2 using the *Speed Lab Design (SLD)* PI gains. Make sure the model step size is set to run at 0.01 seconds. Set the *Amplitude* = 0 and the *Offset* = 35 rad/s. Slowly increase the *Loop*

Gain parameter and add a disturbance by switching the *Disturbance Switch* to the ON position. Record the value that results in a non-decaying oscillation, or just before it goes unstable.

4. Reset the *Loop Gain* to 1 and see how much delay the system can handle by increasing the *nDelay* parameter incrementally and applying a disturbance at each value. Record the value that results in a non-decaying oscillation, or just before it goes unstable.

5. Open the MATLAB script called *DC_Motor_Minimum_Sensitivity_Design.m*. Part of the script is shown below. This script finds the Maximum Sensitivity value, M_s, of the closed-loop function for a range of natural frequencies, ω_n (e.g. 2.5 to 20 rad/s) and plots the results. From this plot, you can find what ω_n results in the smallest value of M_s and, hence, what PI gains are required (from the ω_n and ζ).

   Compare the sensitivity of the idealized model with the model that takes filtering and sampling rate delay into account. Do this by setting the `design_type` parameter to the appropriate value and then run the script. For example, set `design_type=3` to examine both the sampling and filtering effects.

   Attach the sensitivity plots for the ideal plant, the plant with sampling, and the plant with both sampling and filtering. How does the sensitivity change when adding these are taken into account and how will that affect your control design?

```
w = logspace(0,3,1000);
s = 1i * w;
h = 0.01;
% w_ol= 1 / tau;
%
% Transfer function
wf = 50;
Filter = wf ./ ( s + wf );
%
% select effects to add
design_type = 1; % ideal
% design_type = 2; % with sampling
% design_type = 3; % with sampling and filtering
%
iMs = 1;
% find maximum sensitivity value for a range of natural frequencies, wn
for wn = 2.5:0.5:20
    % damping ratio
    zeta= .8;
    % find gains
    kp = ( 2 * zeta * wn * tau - 1 ) / K; % [V.s/rad]
    ki = wn^2 * tau / K;    % [V/rad]
    %
```

6. **Minimum Sensitivity Design (MSD)**: Find the PI gains for the system that includes a sampling rate of 0.01 sec and a filter cut-off frequency of 50 rad/s. Set the damping ratio to 0.8. This will be called the *Minimum Sensitivity Design (MSD)*. Record what the smallest value of M_s is, at what closed-loop natural frequency *wn* this occurs, and the resulting PI gains.

7. Run the model shown in Figure 2.2 using the MSD PI gains found. Similarly as done with the SLD gains, find the *Loop Gain* and *nDelay* required to make the system go into stable / non-decaying oscillations.

8. Present a summary of the results and compare the *Loop Gain* and *nDelay* between SLD and MSD. Which control design is more robust against parameters variations?

9. Stop the QUARC controller and turn off the QUBE-Servo 2 if no more experiments will be conducted.

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