FRICTION IDENTIFICATION

Topics Covered

- Common nonlinear elements found in control systems.
- Friction models and identifying friction parameters.

Prerequisites

- Hardware Interfacing laboratory experiment.
- Filtering laboratory experiment.
- Block Diagram Modeling laboratory experiment.

1 Background

Linear models are often used to characterize the motion of DC servo motors, along with many other systems. While they can be very accurate and adequate for many applications, it is important to know what nonlinearities are neglected during in the modeling process and how they can affect the system response. The major nonlinearities in a motion control system (e.g. servo) are:

- Saturation of the motor amplifier.
- Friction in the motor.
- Quantization of the encoder.

The amplifier that drives the motor has a power supply of 10 V. This means that the voltage from the amplifier can never exceed $V_{max} = 10$ V. A consequence is that the current through the motor is also limited. This limitation implies that the motor transfer function does not describe the system well for large signals.

The other main nonlinearities are due to Coulomb friction and quantization in the encoder. The block diagram of the DC motor shown in Figure 1.1 incorporates the amplifier limitation and the static friction nonlinearities. Encoder quantization and the associated noise was investigated in Filtering laboratory experiment.

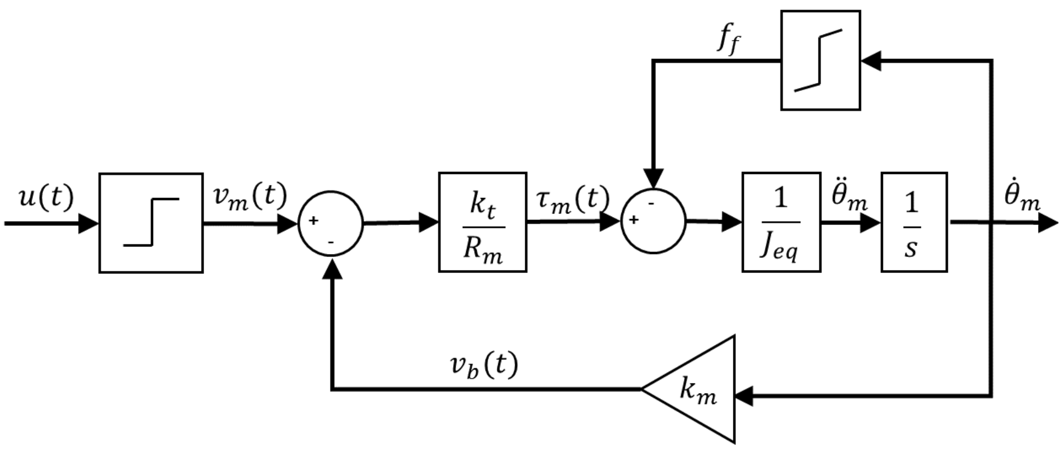


Figure 1.1: DC motor model with amplifier limit and Coulomb and viscous friction

Rotary and linear motion control systems based on mechanical system often have friction. The Coulomb and viscous friction model shown in Figure 1.2 is one of the most common friction models.

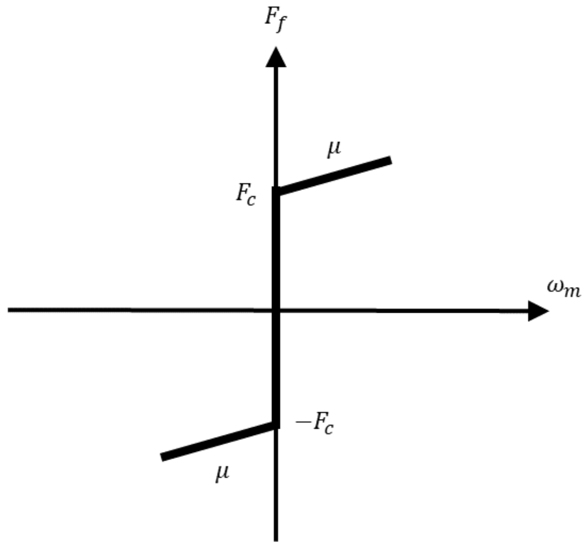


Figure 1.2: Coulomb and viscous friction model

Given the angular velocity of the motor, ω_m, friction can be modeled as follows:

$$F_f = \mu\omega_m + F_c\,\mathrm{sgn}(\omega_m) \tag{1.1}$$

where μ is the viscous friction coefficient and F_c is Coulomb friction parameter. Viscous friction models the dynamic friction and Coulomb friction models the static friction. The force needed to overcome the static friction is known as the *breakaway force*.

The motor shaft equation

$$J_{eq}\dot{\omega}_m(t) = \tau_m(t), \tag{1.2}$$

where J_{eq} is total moment of inertia acting on the motor shaft and τ_m is the applied torque from the DC motor. When including friction, this becomes

$$J_{eq}\dot{\omega}_m(t) = \tau_m(t) - f_f, \tag{1.3}$$

where f_f is represents the torque effects from friction. Based on the current applied, the torque is

$$\tau_m = k_t i_m(t) \tag{1.4}$$

with the motor current

$$i_m(t) = \frac{v_m(t) - k_m\omega_m(t)}{R_m}. \tag{1.5}$$

Applying Equation 1.4 and Equation 1.5 to Equation 1.3 the motion equation becomes

$$J_{eq}\dot{\omega}_m(t) = \frac{k_t}{R_m}(v_m(t) - k_m\omega_m(t)) - f_f \tag{1.6}$$

Since the *back-emf voltage effectively behaves as viscous friction*, we can lump together back-emf and the effects of friction and redefine it in terms of voltage such that

$$J_{eq}\dot{\omega}_m(t) = \frac{k_t}{R_m}(v_m(t) - V_f) \tag{1.7}$$

where

$$V_f = (k_m + \mu_f)\omega_m(t) + V_c\,\mathrm{sgn}(\omega_m). \tag{1.8}$$

The equivalent viscous damping includes the back-emf parameter, k_m, and the additional viscous friction, μ_f, of the motor. The Coulomb friction term V_c is with respect to voltage. This way, we can simplify the model of the DC motor with friction as shown in Figure 1.3.

See Block Diagram Modeling laboratory experimentfor more information about modeling the servo.

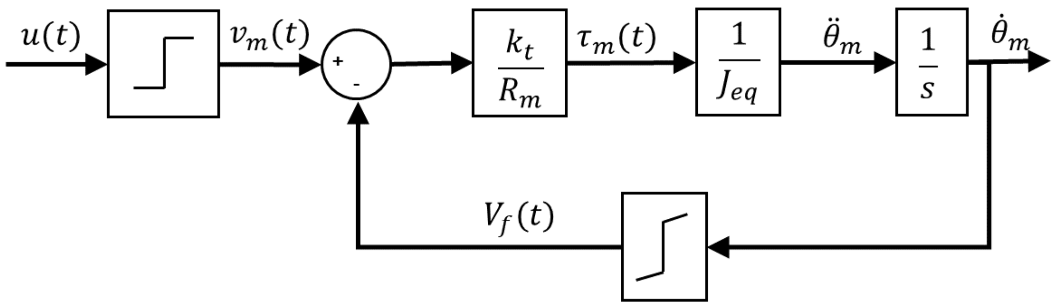


Figure 1.3: DC motor with actuator limits and redefined equivalent friction

2 In-Lab Exercises

2.1 Friction Identification

The Simulink model shown in Figure 2.1 can be used to apply different voltages to the DC motor on the QUBE-Servo 2 and measure its corresponding angular velocity and current. This is used to identify the torque needed to overcome the static friction to get the Coulomb friction parameter as well as find the viscous friction coefficient by looking at the changes in motor velocity with respect to the torque applied.

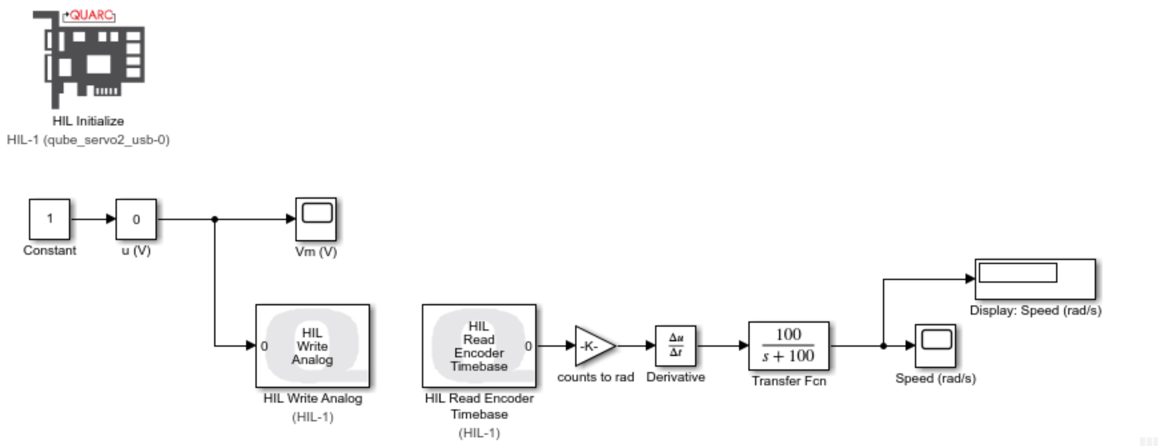


Figure 2.1: Simulink/QUARC model used to identify Coulomb and viscous friction coefficients

1. Create the Simulink model shown in Figure 2.1 using the model designed in Filtering laboratory experiment or open the q_qube2_friction_id Simulink model (if supplied).
2. Set the Slider Gain block to 0 to apply a voltage of 0V to the motor.
3. Build and run the QUARC model.
4. To find the Coulomb friction in the DC motor of the QUBE-Servo 2, gradually increase the voltage applied to the motor in small increments, e.g. 0.01V, until you observe a motion in the inertial disc. Once the inertial wheel starts to rotate (i.e. $\omega_m(t) > 0$), it means you have overcame the static friction. The maximum voltage that still keeps the inertial wheel/DC motor at rest is the positive Coulomb friction voltage V_{c+}.
5. To identify the negative Coulomb friction voltage, V_{c-} start at 0V and repeat the procedure by decreasing the voltage applied to the motor in small increments (e.g. 0.01V).
6. To identify the viscous friction, vary the voltage between -4V and 4V in increments of 1V. Record the resulting angular velocity in a table similarly as shown in Table 2.1.
7. Create a script in MATLAB to generate a plot similarly as shown in Figure 2.2, where the x-axis is the measured velocity and the y-axis is the applied voltage.
8. Use the *Basic Data Fitting* tool in the MATLAB figure to create a linear interpolation of the measured data and display its corresponding equation. Alternatively, you can also use the `polyfit` and `polyval` MATLAB functions to do the interpolation and plotting. The slope of the linear interpolated plot, also given in the equation, is the viscous friction coefficient, μ. **Hint:** Make sure you use at least 3 significant digits.
9. The data collected is based on the total voltage applied and thus, the coefficient found includes the back-emf as well as friction from other sources. Given the back-emf parameter, k_m, is already known, find what the viscous coefficient of friction is without this.
10. Stop the QUARC controller and turn off the QUBE-Servo 2 if no more experiments will be conducted.

Applied Voltage (V)	Measured Velocity (rad/s)
-4	
-3	
-2	
-1	
V_{c-}	0
0	0
V_{c+}	0
-1	
-2	
-3	
-4	

Table 2.1: Friction identification measurements

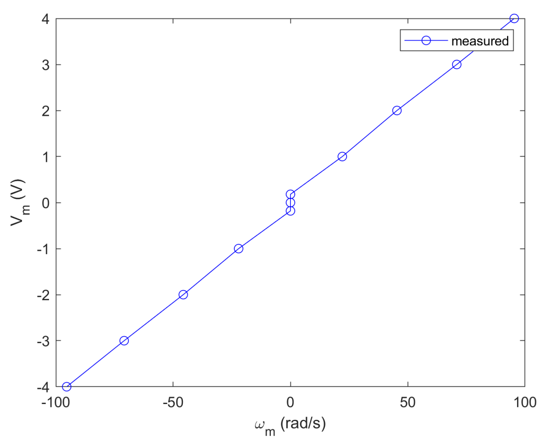


Figure 2.2: Measured Coulomb and viscous friction

2.2 Model Friction Validation

In Block Diagram Modeling laboratory experiment, a block diagram was designed to represent the DC motor of the QUBE-Servo 2. This was a linear model that did not include the nonlinear Coulomb friction effects, or the additional viscous friction (i.e. in addition to the back-emf, which was already modeled).

1. Open the *q_qube2_friction_model_student* Simulink model shown in Figure 2.3.
2. Open and run the `qube2_param.m` MATLAB script to load the various DC motor parameters.
3. Using the diagram of the QUBE-Servo 2 shown in Figure 1.1 and the various equations, complete the block diagram in Simulink, similarly as done in Block Diagram Modeling laboratory experiment using Gain, Subtract, and Integrator blocks. Part of the solution is shown in Figure 2.3. Attach a screen capture of the completed Simulink model.
   (a) Use the Coulomb & Viscous Friction block found in the *Discontinuous* library to model the friction. As shown in Figure 1.2, the Coulomb friction model assumes the positive and negative force, F_c, to be symmetrical. If the positive and negative Coulomb friction found are not the same, i.e. $V_{c+} \neq V_{c-}$, take the largest one and let that be the equivalent Coulomb friction, i.e.$V_c =$ max $\{V_{c+}, V_{c-}\}$.

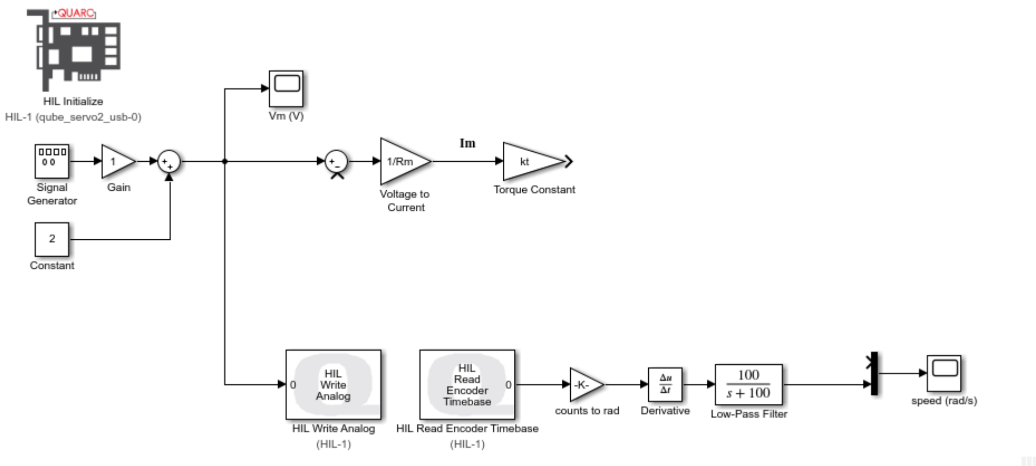
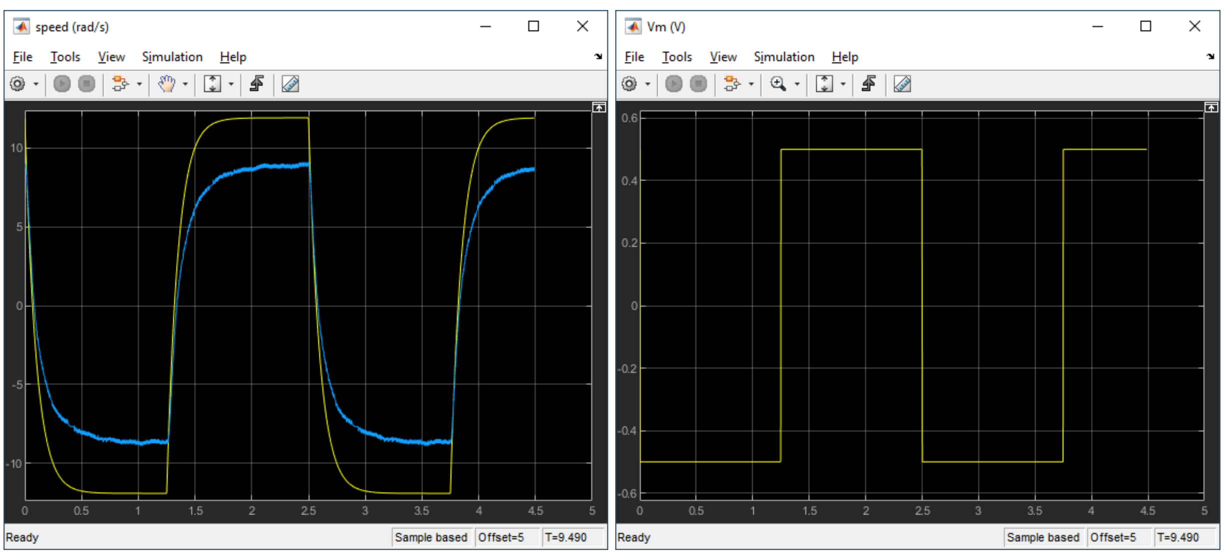


Figure 2.3: Simulink/QUARC model used to identify Coulomb and viscous friction coefficients

(b) When defining the values in the Gain blocks, use the MATLAB variable names that are defined in the `qube2_param.m` script.

4. Set the model to apply an input voltage of ± 0.5 V (i.e. 0 V offset).

5. Build and run the model in QUARC. The sample response shown in Figure 2.4 shows the input motor voltage and the measured and simulated response of the QUBE-Servo 2 *without any friction modeled*.



(a) Velocity

(b) Voltage

Figure 2.4: Example model validation response using block diagram model *without friction*

6. Apply the Coulomb and viscous friction parameters found in 2.1 and run the Simulink/QUARC model. Attach the response you obtained when the friction terms are included. **Hint:** When defining the viscous friction in the Coulomb & Viscous Friction block, make sure you use the viscous coefficient is the one that includes both the back-emf and friction effects (i.e. the total friction).

7. Does including friction improve the accuracy of the model? Explain your results and the procedure you used to validate this.

8. Stop the QUARC controller and power off the QUBE-Servo 2 if no more experiments will be performed.

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