

# STATE-SPACE MODELING

## Topics Covered

- Modeling from first-principles.
- State-space representation.
- Model validation.

## Prerequisites

- Hardware Interfacing laboratory experiment.
- Filtering laboratory experiment.

# 1 Background

## 1.1 Linear State-Space Representation

The standard state-space representation of a multi-input multi-output (MIMO) continuous linear-time invariant (LTI) system with  $n$  state variables,  $r$  input variables, and  $m$  output variables is

$$\dot{x}(t) = Ax + Bu \quad (1.1)$$

$$y(t) = Cx(t) + Du(t) \quad (1.2)$$

where  $x$  is the vector of state variables ( $n \times 1$ ),  $u$  is the control input vector ( $r \times 1$ ),  $y$  is the output vector ( $m \times 1$ ),  $A$  is the system matrix ( $n \times n$ ),  $B$  is the input matrix ( $n \times r$ ),  $C$  is the output matrix ( $m \times n$ ), and  $D$  is the feed-forward matrix ( $m \times r$ ).

The block diagram representation of state-space Equation 1.1 and Equation 1.2 is shown in Figure Figure 1.1.

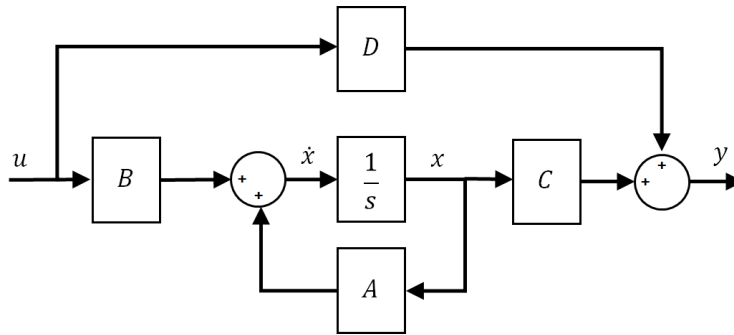


Figure 1.1: State-Space Block Diagram

## 1.2 DC Motor Modeling

This section summarizes how to find the equations of motion (EOMs) of the DC motor. The motor electrical equation is

$$v_m(t) - R_m i_m(t) - k_m \dot{\theta}_m(t) = 0 \quad (1.3)$$

where  $v_m(t)$  is the motor input voltage (the control input),  $R_m$  is the motor electrical resistance,  $i_m(t)$  is the current,  $k_m$  is the back-emf constant, and  $\theta_m(t)$  is the angular position of the motor shaft (i.e. the inertia disc).

The motor shaft equation is expressed as

$$J_{eq} \ddot{\theta}(t) = \tau_m(t) \quad (1.4)$$

where  $J_{eq}$  is the total or equivalent moment of inertia acting on the motor shaft and  $\tau_m$  is the applied torque from the DC motor. Based on the current applied, the torque is

$$\tau_m(t) = k_t i_m(t) \quad (1.5)$$

where  $k_t$  is the motor current torque constant.

See Block Diagram Modeling laboratory experiment for more information about finding the servo equations of motion.

## 2 In-Lab Exercises

The **SIMULINK**<sup>®</sup> model shown in Figure 2.1 applies a 1 V step to the QUBE-Servo 2 hardware, using **QUARC**<sup>®</sup>, and the state-space model of the servo. The measured and simulated speed response (i.e. from the model) are plotted in the same scope to compare.

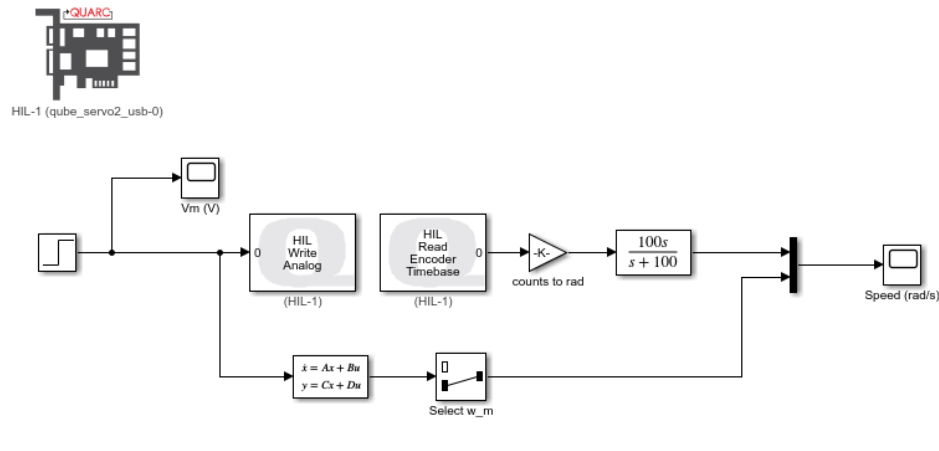
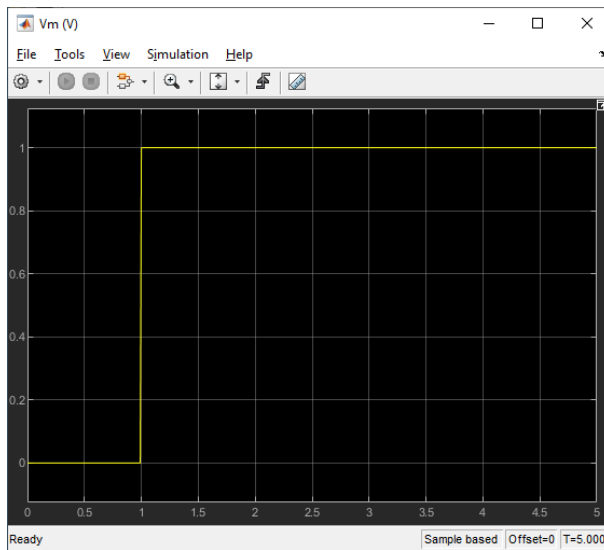
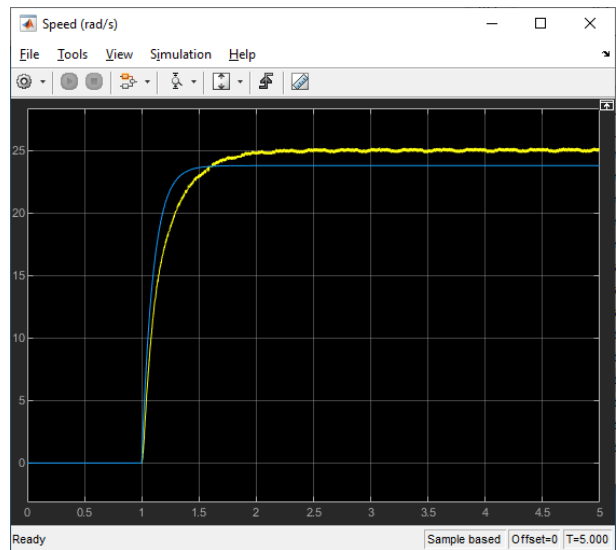


Figure 2.1: Applies a step voltage and displays measured and simulated DC motor response.

1. Formulate the differential equation relating the motor position,  $\theta_m$ , and the motor input voltage,  $v_m(t)$ , using Equation 1.3, Equation 1.4, and Equation 1.5.
2. Derive the state-space model of the DC motor from the differential equation you obtained in Exercise 1 for the following state variables:  $x_1 = \theta_m(t)$  and  $x_2 = \dot{\theta}_m(t)$ ,  $y_1 = \theta_m(t)$  and  $y_2 = \dot{\theta}_m(t)$  (i.e. measuring motor position and speed), and the input variable  $u = v_m(t)$ .
3. Based on the state space model derived in Step 2 and using the parameters defined in the the **MATLAB**<sup>®</sup> script `qube2_param.m` provided, create the **MATLAB**<sup>®</sup> script that constructs a MATLAB state-space model and generate its step response.
4. Open or design the Simulink model shown in Figure 2.1. This applies a 1 V step to the QUBE-Servo 2 system and its state-space model. Use the model already designed in the Filtering laboratory experiment if Figure 2.1 is to be designed.
5. Run the **MATLAB**<sup>®</sup> script created in Exercise 3 to load the state space model parameters in the **MATLAB**<sup>®</sup> workspace. Ensure that the generated matrices match your solution in Step 2.
6. Build and run the model. The scope response should be similar to Figure 2.2. Attach a screen capture of your scopes. Does your model represent the actual DC motor well?



(a) Input Voltage (V)



(b) Motor Angular Rate (rad/s)

Figure 2.2: Step response of the DC motor and State-Space Model

7. Stop the **QUARC**<sup>®</sup> controller.

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