LEAD COMPENSATOR

Topics Covered

- Lead Compensator Design
- Bode plots

Prerequisites

- Hardware Interfacing laboratory experiment
- Filtering laboratory experiment

1 Background

PID control uses integration and derivative operations to design a compensator such that the overall system response follows a desired trajectory. More generally, the design of a controller for a system can be regarded as a filter design problem. In this context, a PI controller is a *low-pass filter*, and because it introduces a negative phase over some frequency range, therefore it is also called a **phase lag controller**. Contrary, a PD controller is a *high-pass filter* that introduces a positive phase and is also referred to as a **phase lead controller**.

The generic transfer function of a simple *lead or lag* compensator can be expressed as

$$C(s) = K_c \frac{s+z}{s+p} = K_c \frac{1+\alpha T s}{1+Ts} \tag{1.1}$$

This becomes a *phase lag* controller, i.e low-pass, when $\alpha < 1$ (or $p < z$) and a *phase lead* controller, i.e. high-pass, when $\alpha > 1$ (or $p > z$).

The proportional gain of the lead compensator is used to attain a certain crossover frequency. In general, increasing the gain, and respectively the crossover frequency, essentially increases the bandwidth of the system, thus decreasing the system's peak time (speeding up the response). A gain of $K_c > 1$ decreases the system's phase margin and, if K_c is chosen too large, will lead to large overshoots in the system response. For design purposes, K_c is often chosen such that it increases the bandwidth of the system to about half the desired bandwidth. The lead compensator will add additional gain such that the combination of K_c and lead compensator result in the desired system bandwidth.

Even though lag compensators work well in theory, they often struggle with the saturation limits of actual hardware, and may not be able to achieve a zero steady-state error specification. In this lab, we will design a lead compensator in series with an integrator as in Figure 1.1 to achieve zero steady-state error. The resulting controller has the form

$$C(s) = K_c \frac{1+\alpha T s}{(1+Ts)s}. \tag{1.2}$$

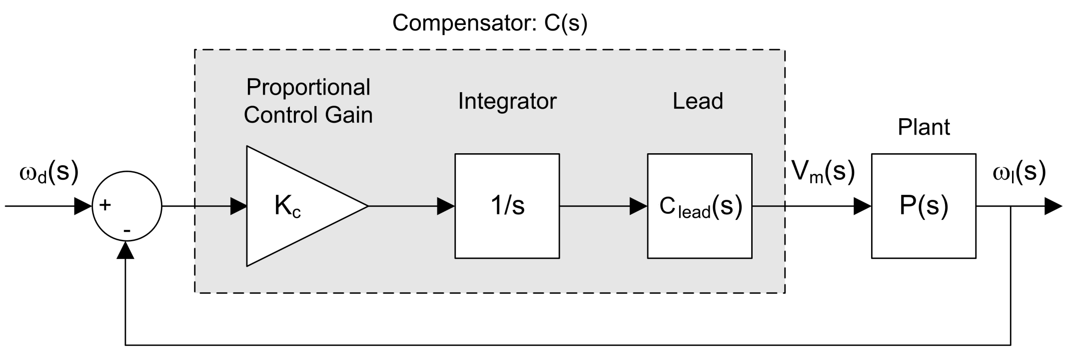


Figure 1.1: Closed-loop speed control with lead compensator

1.1 Lead compensator design procedure

The two main design parameters for a lead compensator are the phase margin and the gain crossover frequency. The phase margin determines how much delay the system can withstand before going unstable and it tends to affect the overshoot of the response. A higher phase margin implies a more stable response with less overshoot. Generally speaking, the percent overshoot, PO, of a system should not go beyond 5 % for a phase margin of at least 75 deg. This can be used as a guideline for the initial control design.

The gain crossover frequency, ω_m, is defined as the frequency where the gain of the system is 1 (or 0 dB in a Bode plot). This parameter mainly affects the speed of the response and a larger ω_m implies a decrease in the peak time. Generally speaking, the peak time t_p should not exceed 0.05 s when the gain crossover frequency is at least 75 rad. This can be used as a guideline for the initial control design.

The design process for a lead compensator can be summarized as follows:

1. Generate the Bode plot of the open-loop uncompensated system.
2. The lead compensator itself will add some gain to the closed-loop system response. To make sure that the bandwidth requirement of the design can be met, a proportional gain K_c needs to be added such that the open-loop crossover frequency is about a factor of two below the desired system bandwidth.
3. Determine the necessary additional phase lead ϕ_m for the plant with open-loop gain K_c. To do so, compute

$$\phi_m = PM_{des} - PM_{meas}, \tag{1.3}$$

4. Compute α. To attain the maximum phase ϕ_m at the frequency ω_m as shown in Figure 1.2, the compensator is required to add $20\,\log_{10}(\alpha)$ of gain. Here, ω_m is the geometric mean of the two corner frequencies from the zero and pole of the lead compensator, respectively, i.e.

$$\log_{10}(\omega_m) = \frac{1}{2}\left(\log_{10}\left(\frac{1}{\alpha T}\right) - \log_{10}\left(\frac{1}{T}\right)\right). \tag{1.4}$$

Solving for ω_m reveals

$$\omega_m = \frac{1}{T\sqrt{\alpha}}. \tag{1.5}$$

The proportional gain of the lead compensator is used to attain a certain crossover frequency. In general, increasing the gain, and respectively the crossover frequency, essentially increases the bandwidth of the system, thus decreasing the system's peak time (speeding up the response). A gain of $K_c > 1$ decreases the system's phase margin and, if K_c is chosen too large, will lead to large overshoots in the system response.

The lead compensator is used to dampen the overshoot and increase the overall stability of the system by increasing the phase margin. The frequency response of the lead compensator in (Equation 1.1) is given by substituting $s = j\omega$ as

$$C_{lead}(j\omega) = \frac{1 + j\omega\alpha T}{1 + j\omega T}, \tag{1.6}$$

with the corresponding magnitude and phase

$$\begin{aligned} |C_{lead}(j\omega)| &= \sqrt{\frac{1+\omega^2\alpha^2T^2}{1+\omega^2T^2}}, \\ \phi_m &= \arctan\left(\omega\alpha T\right) - \arctan\left(\omega T\right). \end{aligned} \tag{1.7}$$

Using the trigonometric identity

$$\tan\left(\alpha - \beta\right) = \frac{\tan\left(\alpha\right) + \tan\left(\beta\right)}{1 + \tan\left(\alpha\right)\tan\left(\beta\right)}$$

on Equation 1.7 yields

$$\tan\left(\phi_m(j\omega)\right) = \frac{\omega\alpha T - \omega T}{1 + (\omega\alpha T)(\omega T)}. \tag{1.8}$$

Noting

$$\tan\left(\alpha\right) = \pm\frac{\sin\left(\alpha\right)}{\sqrt{1 - \sin^2\left(\alpha\right)}},$$

and using Equation 1.5, one can find

$$\sin\left(\phi_m\right) = \frac{\alpha - 1}{\alpha + 1}. \tag{1.9}$$

Thus, if ϕ_m is known, α can be determined by solving

$$\alpha = \frac{1 + \sin\left(\phi_m\right)}{1 - \sin\left(\phi_m\right)}. \tag{1.10}$$

5. Determine the value of T using Equation 1.5. To do so, place the corner frequencies of the lead compensator such that ϕ_m is located at ω_m, i.e. the new gain crossover frequency (the geometric mean of $1/\alpha T$ and $1/T$) where the compensator has a gain of 10 dB. By design, ω_m is the frequency at which the system with compensator has 0 dB gain. Therefore, ω_m has to be placed at the frequency where the magnitude of the uncompensated system is $C(j\omega) = -10\log_{10}\alpha$ dB. ω_m is then obtained by finding the corresponding frequency in the uncompensated Bode plot.

6. Determine the pole and zero of the lead compensator.

7. Check whether or not the compensator fulfills the design requirements. To do so, draw the Bode plot of the compensated system and check the resulting phase margin and check whether or not the system response meet the desired characteristics. Repeat the design steps for a different ϕ_m if necessary.

A typical Bode plot of a lead compensator is shown in Figure 1.2.

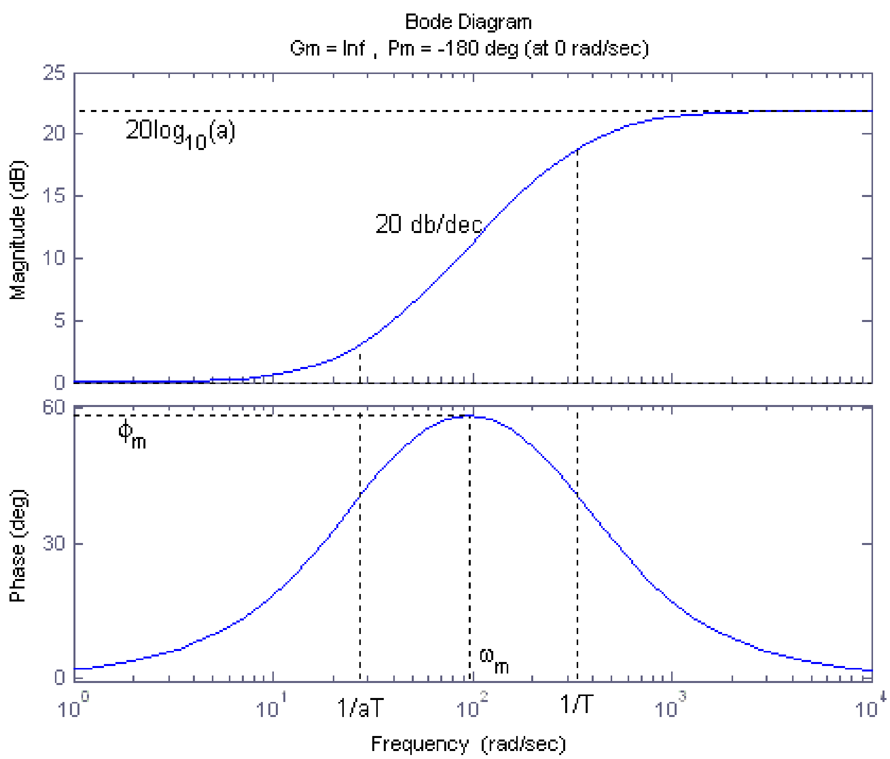


Figure 1.2: Bode plot of a typical lead compensator

2 In-Lab Exercises

In this lab, you will design a lead compensator for the speed control of the QUBE-Servo 2. Recall that the input voltage to output speed transfer function for the QUBE-Servo 2is given by

$$P(s) = \frac{K}{\tau s + 1}. \tag{2.1}$$

As stated in the background section, we want to design a controller that is in series with an integrator to guarantee zero steady-state. For the design purpose of the lead compensator, we assume that the integrator is part of the plant model, i.e.

$$P_i(s) = P(s)\frac{1}{s}. \tag{2.2}$$

The control design should fulfill the following design requirements for steady-state error (e_{ss}), peak time (t_p), percentage overshoot (PO), phase margin (PM) and system bandwidth (ω_m):

$$\begin{aligned} e_{ss} &= 0, \\ t_p &= 0.05 \text{ s}, \\ PO &\leq 7.5 \text{ \%}, \\ PM &\geq 85 \text{ deg}, \\ \omega_m &\geq 45.0 \text{ rad/s}. \end{aligned} \tag{2.3}$$

2.1 Lead Compensator Design

1. Find the magnitude of the frequency response of the system transfer function Equation 2.2 that is in series with an integrator ($|P_i(s)|$) in terms of the frequency ω.
2. The system has a gain of 1 (or 0 dB) at the crossover frequency ω_g. Find an expression for the crossover frequency in terms of the model parameters K and τ for $P_i(s)$. Use this expression to determine the crossover frequency for the QUBE-Servo 2 using the default model parameters $K = 22.4$ and $\tau = 0.15$ or, alternatively, by using more precise values found by going through one of the modeling experiments, e.g. Step Response Modeling laboratory experiment.
3. Generate the Bode plot of $P_i(s)$ using the `margin(Pi)` command. Compare your derived crossover frequency from the previous step to that obtained using `margin(Pi)`.

   Hint: The MATLAB command `Pi = tf(num,den)` generates a transfer function object in the workspace, where `num` and `den` are vectors of the numerator and denominator polynomials of `Pi`, respectively. The MATLAB command `margin(Pi)` generates a Bode plot and lists the gain margin and phase margin with their respective crossover frequencies.

4. Find the proportional gain K_c that is necessary such that $K_c P_i(s)$ has a crossover frequency of 20 rad/s (less than half the desired closed-loop bandwidth).
5. Determine the necessary phase lead, ϕ_m, that the lead compensator needs to add for the system $K_c P_i(s)$.
6. Compute α.
7. Determine the gain crossover frequency, ω_m.
8. Does ω_m meet the design requirement? Comment on what you could do to ensure you meet this requirement.
9. Determine the transfer function of the lead compensator. Start by evaluating T.
10. Determine the pole and zero location of the lead compensator. Generate the Bode plot of your lead compensator and verify that you have the desired phase margin at the desired frequency.

11. Validate your result by obtaining the bode plot of the system loop transfer with the proportional gain K_c and lead compensator $C_L(s)$ in series with the plant. Do you have the desired phase margin at the desired frequency?

12. Simulate the step response of the closed-loop system in MATLAB using the `step` function to verify if the time-domain requirements are satisfied. Attach the code used to generate the step response as well as the response itself.

2.2 Lead Compensator Implementation

The Simulink model shown in Figure 2.1 implements the lead compensator designed on the QUBE-Servo 2 using QUARC®.

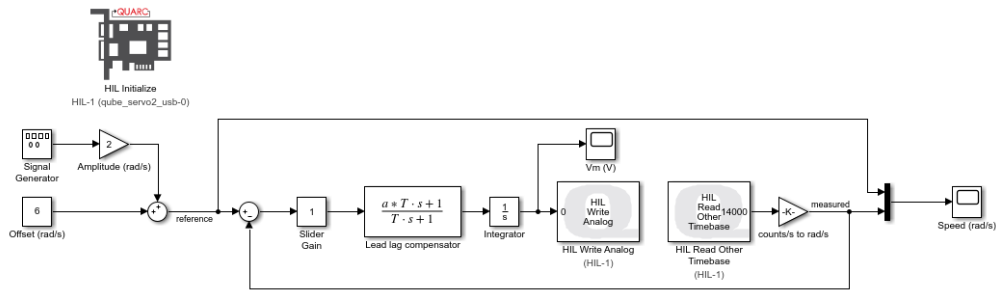
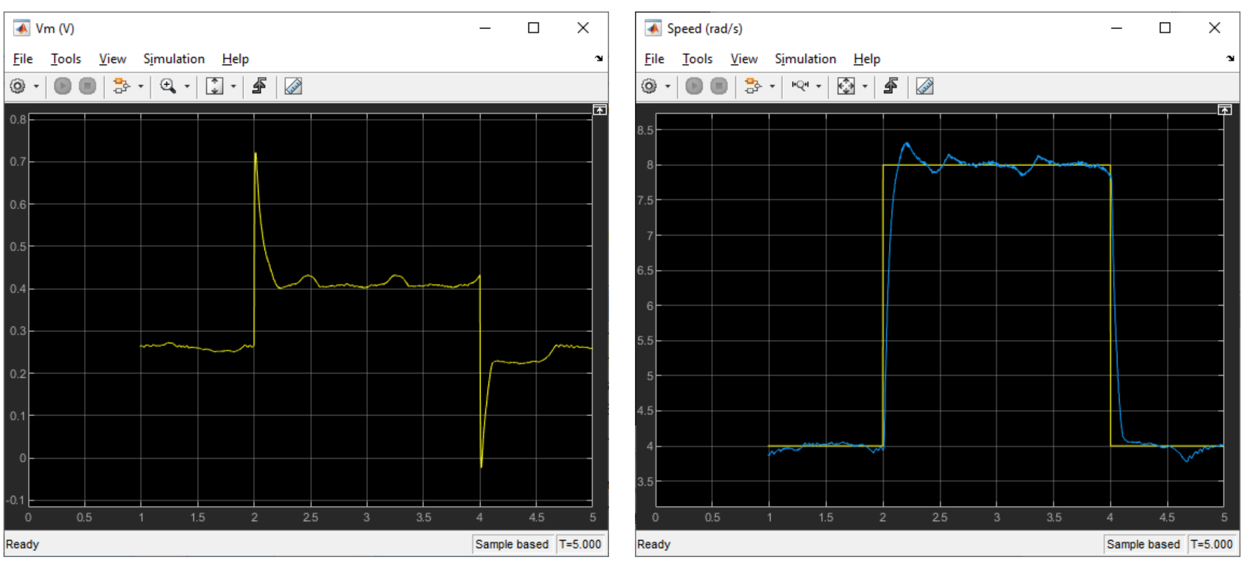


Figure 2.1: Implementation of lead compensator on QUBE-Servo 2

1. Open the `q_qube2_lead` shown in Figure 2.1 (if supplied), or design your own.
2. Set the `Slider Gain` block to the proportional gain found and ensure the `a` and `T` lead compensator variables are defined in the MATLAB® workspace based on your lead compensator design.
3. Build and run the model in QUARC®. A sample response is shown in Figure 2.2.



(a) Voltage (V)

(b) Speed (rad/s)

Figure 2.2: Example speed control response using lead compensator with $K_c = 1$

4. Attach the response you obtained.

5. Does the system response match the desired time-domain characteristics? If not, try varying the value of K_c and see if you can improve the overall system response.

   Hint: Use the *Cursor Measurements* tool in the SIMULINK® Scope toolbar to obtain your measurements.

6. Make sure to stop you QUARC® model (if you were running it continuously).

7. Power off the QUBE-Servo 2 if no more experiment will be conducted in this session.

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