BLOCK DIAGRAM MODELING

Topics Covered

- First-principles modeling
- Block diagram representation
- Transfer function model
- Model validation

Prerequisites

- Hardware Interfacing laboratory experiment.
- Filtering laboratory experiment.

1 Background

The Quanser QUBE-Servo 2 is a direct-drive rotary servo system. Its motor armature circuit schematic is shown in Figure 1.1 and the electrical and mechanical parameters are given in Table 1.1. The DC motor shaft is connected to the *load hub*. The hub is a metal disk used to mount the disk or rotary pendulum and has a moment of inertia of J_h. A disk load is attached to the output shaft with a moment of inertia of J_d.

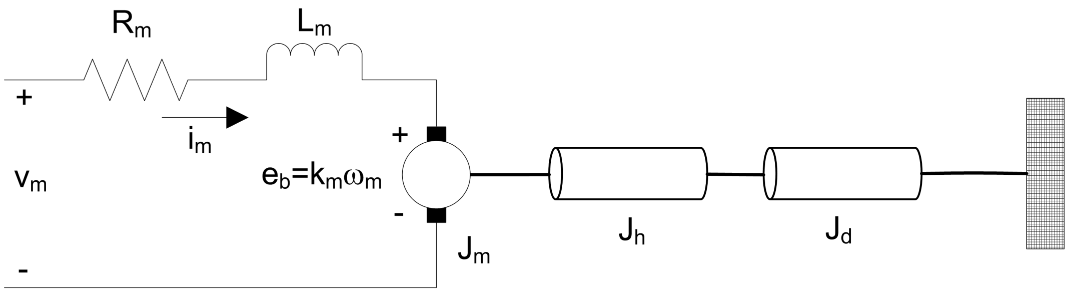


Figure 1.1: QUBE-Servo 2 DC motor and load

The back-emf (electromotive) voltage $e_b(t)$ depends on the speed of the motor shaft, ω_m, and the back-emf constant of the motor, k_m. It opposes the current flow. The back emf voltage is given by:

$$e_b(t) = k_m \omega_m(t) \tag{1.1}$$

Symbol	Description	Value
DC Motor		
R_m	Terminal resistance	8.4Ω
k_t	Torque constant	0.042 N.m/A
k_m	Motor back-emf constant	0.042 V/(rad/s)
J_m	Rotor inertia	4.0×10^{-6} kg.m^2
L_m	Rotor inductance	1.16 mH
m_h	Load hub mass	0.0106 kg
r_h	Load hub mass	0.0111 m
J_h	Load hub inertia	0.6×10^{-6} kg.m^2
Load Disk		
m_d	Mass of disk load	0.053 kg
r_d	Radius of disk load	0.0248 m

Table 1.1: QUBE-Servo 2 system parameters

Using Kirchoff's Voltage Law, we can write the following equation:

$$v_m(t) - R_m i_m(t) - L_m \frac{di_m(t)}{dt} - k_m \omega_m(t) = 0. \tag{1.2}$$

Since the motor inductance L_m is much less than its resistance, it can be ignored. Then, the equation becomes

$$v_m(t) - R_m i_m(t) - k_m \omega_m(t) = 0. \tag{1.3}$$

Solving for $i_m(t)$, the motor current can be found as:

$$i_m(t) = \frac{v_m(t) - k_m \omega_m(t)}{R_m}. \tag{1.4}$$

The motor shaft equation is expressed as

$$J_{eq} \dot{\omega}_m(t) = \tau_m(t), \tag{1.5}$$

where J_{eq} is total moment of inertia acting on the motor shaft and τ_m is the applied torque from the DC motor. Based on the current applied, the torque is

$$\tau_m = k_t i_m(t) \tag{1.6}$$

The moment of inertia of a disk about its pivot, with mass m and radius r, is

$$J = \frac{1}{2} m r^2. \tag{1.7}$$

2 In-Lab Exercises

Based on the models already designed in Hardware Interfacing and Filtering laboratory experiment, design a model that applies a $1-3$ V, 0.4 Hz square wave to the motor and reads the servo velocity using the encoder as shown in Figure 2.1.

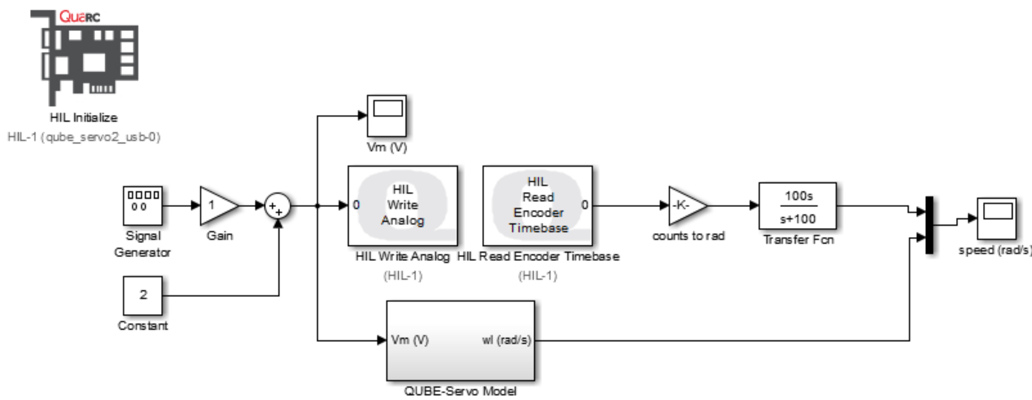


Figure 2.1: Applies a step voltage and displays measured and simulated QUBE-Servo 2 speed.

Create subsystem called *QUBE-Servo 2 Model*, as shown in Figure 2.1, that contains blocks to model the QUBE-Servo 2 system. Thus using the equations given above, assemble a simple block digram in SIMULINK® to model the system. You'll need a few Gain blocks, a Subtract block, and an Integrator block (to go from acceleration to speed). Part of the solution is shown in Figure 2.2.

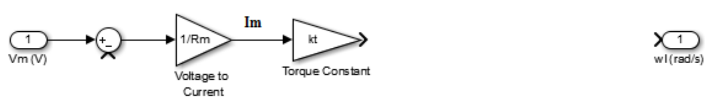


Figure 2.2: Incomplete *QUBE-Servo 2 Model* subsystem.

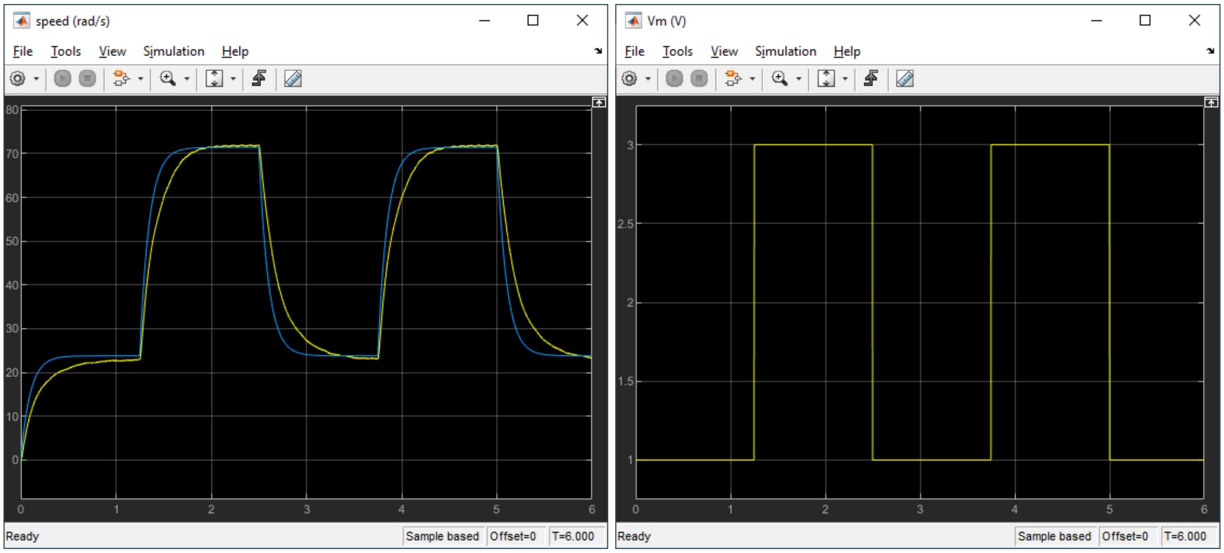
It may also help to write a short MATLAB® script that sets the various system parameters in MATLAB®, so you can use the symbol instead of entering the value numerically in the Gain blocks. In the example shown in Figure 2.2, we are using *Rm* for motor resistance and *kt* for the current-torque constant. To define these, write a script like:

```
% Resistance
Rm = 8.4;
% Current-torque (N-m/A)
kt = 0.042;
```

1. The motor shaft of the QUBE-Servo 2 is attached to a *load hub* and a disk load. Based on the parameters given in Table 1.1, calculate the equivalent moment of inertia that is acting on the motor shaft.

2. Complete the block diagram model of the QUBE-Servo 2 and place it the SIMULINK® *QUBE-Servo 2 Model* subsystem similarly as shown in Figure 2.2. Attach a screen capture of your completed block diagram model and a MATLAB® script that defines the system parameters.

3. Build and run the QUARC controller with your QUBE-Servo 2 model. The scope response should be similar to Figure 2.3. Attach a screen capture of your scopes. Does your model represent the QUBE-Servo 2 reasonably well?



(a) Motor Speed

(b) Motor Voltage

Figure 2.3: Example response when running block diagram model and with QUBE-Servo 2 hardware in parallel

4. You may notice that the the model does not match the measured system exactly. What could cause this difference? Given on possible source.
5. Formulate the differential equation for ω_m using Equation 1.4 to Equation 1.6.
6. Take the Laplace Transform and find the voltage to speed transfer function, $\Omega(s)/V_m(s)$, of the system. Evaluate the transfer function numerically.
7. Stop the QUARC® controller.
8. Power off the QUBE-Servo 2.

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