LOAD DISTURBANCE

Topics Covered

- Proportional-derivative-integral (PID) compensator.
- Finding closed-loop transfer function.
- Effect of load disturbance.
- Evaluating steady-state angle.

Prerequisites

- Integration laboratory experiment.
- Filtering laboratory experiment.
- First Principles Modeling laboratory experiment.
- PD Control laboratory experiment.

1 Background

The closed-loop position control system of the QUBE-Servo 2 using PID can be represented by the block diagram shown in Figure 1.1. The motor has motor voltage V_m and torque T_d as inputs and motor angle θ_m as the output. The torque is typically a disturbance torque that you apply manually to the inertial load. V_{sd} is a simulated external disturbance voltage that is applied through software.

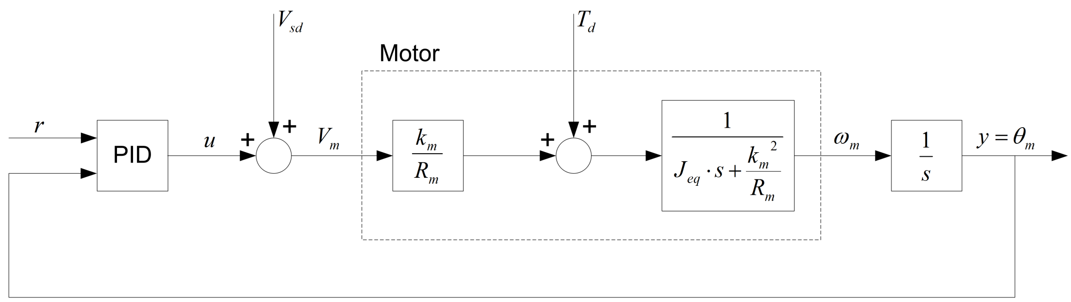

Figure 1.1: QUBE-Servo 2 PID closed-loop position control

1.1 DC Motor Model

The QUBE-Servo 2 voltage-to-position transfer function is

$$P(s) = \frac{\Theta_m(s)}{V_m(s)} = \frac{K}{s(\tau s + 1)}, \tag{1.1}$$

where $K = 21.9$ rad/(V − s) is the model steady-state gain, $\tau = 0.15$ s is the model time constant, $\Theta_m(s) = \mathcal{L}[\theta_m(t)]$ is the motor / disk position, and $V_m(s) = \mathcal{L}[v_m(t)]$ is the applied motor voltage. If desired, you can conduct an experiment to find more precise model parameters, K and τ, for your particular servo (e.g. performing the Bump Test Modeling or the First Principles Modeling laboratory experiment).

Recall that k_m is the motor back-emf constant, R_m is the motor resistance, and J_{eq} is the equivalent moment of inertia that was found in First Principles Modeling laboratory experiment.

1.2 PID Control

The proportional, integral, and derivative control can be expressed mathematically as follows

$$u(t) = k_p e(t) + k_i \int_0^t e(\tau) d\tau + k_d \frac{de(t)}{dt}. \tag{1.2}$$

The control action is a sum of three terms referred to as proportional (P), integral (I) and derivative (D) control gain. The controller Equation 1.2 can also be described by the transfer function

$$C(s) = k_p + \frac{k_i}{s} + k_d s. \tag{1.3}$$

Given the relationship

$$\frac{R_m}{k_m} = \frac{\tau}{K J_{eq}}$$

and assuming the simulated disturbance voltage is zero, $V_{sd} = 0$, and no reference, $r = 0$, the block diagram can be reduced to Figure 1.2.

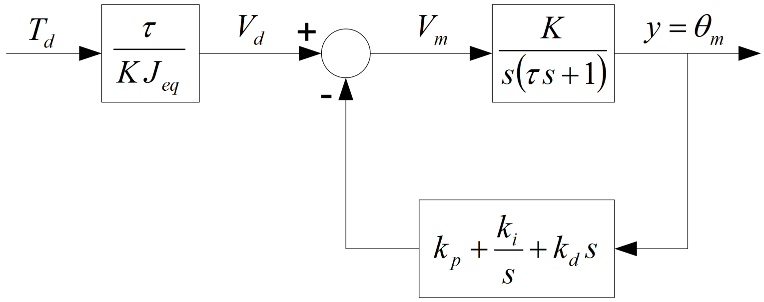

Figure 1.2: QUBE-Servo 2 PID disturbance regulation

1.3 Final Value Theorem

The Final Value Theorem can be used to determine the steady-state or final value of the system output $y(t)$ given its Laplace transform $Y(s)$. For a system with an unstable pole (i.e. a pole in the right half of the s-plane), the final value is unbounded. If the system is marginally stable with a complex conjugate pole pair on the imaginary axis, the output of the system will be oscillatory and the final value is not defined. For a stable system response (i.e. all poles of the system are strictly in the left half of the s-plane), the following holds

$$\lim_{s\to 0} sY(s) = \lim_{t\to\infty} y(t). \tag{1.4}$$

2 In-Lab Exercises

Based on the model in the QUBE-Servo 2 PD Control laboratory experiment, design a QUARC controller that controls the position of the servo using a PID compensator and can apply a simulated voltage, V_{sd}, to the motor. See the example given in Figure 2.1 as well as the full block diagram in Figure 1.1.

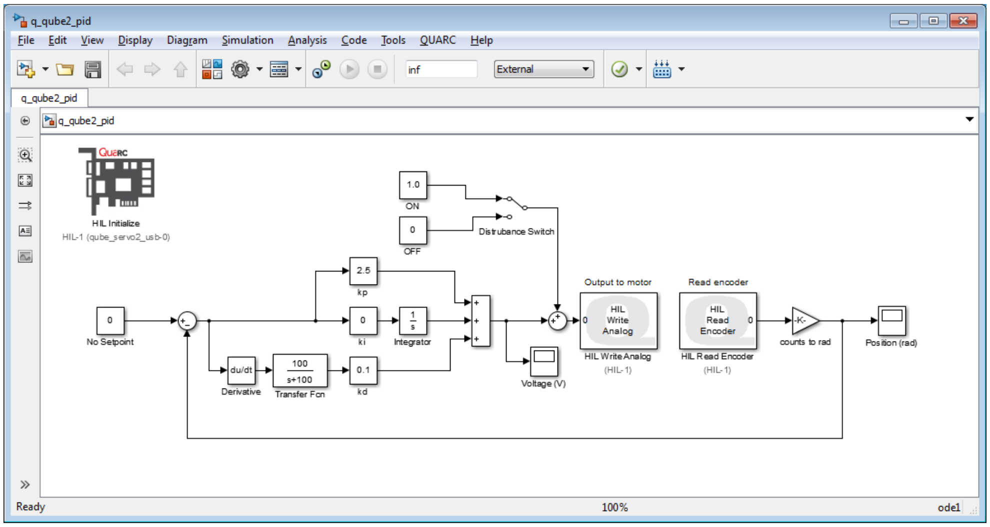

Figure 2.1: Implements PID position control and adds simulated disturbance

1. Find the closed-loop transfer function from disturbance torque to motor angle

$$G_{\theta,T}(s) = \frac{\Theta_m(s)}{T_d(s)}$$

   as a function of the system parameters k_p, k_i, k_d, K, τ, and J_{eq}.

2. Find the steady-state angle of the QUBE-Servo 2 when using PD control with gains $k_p = 2.5$ and $k_d = 0.01$ and with the full PID with gain $k_i = 0.1$ for the following load disturbance:

$$T_d = \frac{T_{d0}}{s}. \tag{2.1}$$

   Explain how the different terms in the PID control (i.e. proportional, integral, and derivative) effect how a load disturbance is handled.

   Hint: Use the final value theorem (Equation 1.4) to determine the steady state angle.

3. Assume that a constant simulated disturbance voltage $V_{sd} = V_{d0} = 1$ V is added to the motor voltage to simulate a disturbance torque $T_d = T_{d0}$. Using the model parameters and the equivalent moment of inertia, J_{eq}, found in First Principles Modeling laboratory experiment determine and evaluate the corresponding disturbance torque amplitude T_{d0}. Evaluate then the resulting steady-state angle $\theta_{ss,pd}$ using a proportional gain of $k_p = 2.5$ V/rad and derivative gain $k_d = 0.1$.

4. Connect and power up the QUBE-Servo 2 system and ensure the inertial disc load is mounted.

5. Open the QUARC controller shown in Figure 2.1 to experimentally evaluate the steady-state angle of the QUBE-Servo 2 when a simulated disturbance voltage is applied. Use the QUARC controller to apply a simulated disturbance of $V_{sd} = 1$ V when using a PD controller with gains $k_p = 2.5$ and $k_d = 0.1$. Attach the position response as well as the motor voltage used.

6. Compare the experimental result with the one found in Question 3. If there were any discrepancies between the theoretical and experimental results, explain what could be the cause.

7. Try varying the proportional gain. What effect does it have on the steady-state angle when a disturbance is added?
8. Re-run the QUARC controller when using a PID controller with the gains $k_p = 2.5$, $k_d = 0.1$, and $k_i = 1.5$. Attach a response showing the resulting steady-state angle and the motor voltage. Does this match your expected result?
9. How does the integral gain effect the position response when a disturbance is applied?
10. Stop the QUARC controller and turn off the QUBE-Servo 2 if no more experiments will be conducted.

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