

# PARAMETER ESTIMATION

## Topics Covered

- Obtaining the equations of motion of a DC motor based rotary servo.
- Transfer function modeling.
- Modeling from experimental tests - parameter estimation.

## Prerequisites

- Integration laboratory experiment.
- Filtering laboratory experiment.
- First Principles Modeling laboratory experiment.

# 1 Background

The Quanser QUBE-Servo 2 is a direct-drive rotary servo system. Its motor armature circuit schematic is shown in Figure 1.1 and the electrical and mechanical parameters are given in Table 1.1. The DC motor shaft is connected to the *load hub*. The hub is a metal disk used to mount the disk or rotary pendulum and has a moment of inertia of  $J_h$ . A disk load is attached to the output shaft with a moment of inertia of  $J_d$ .

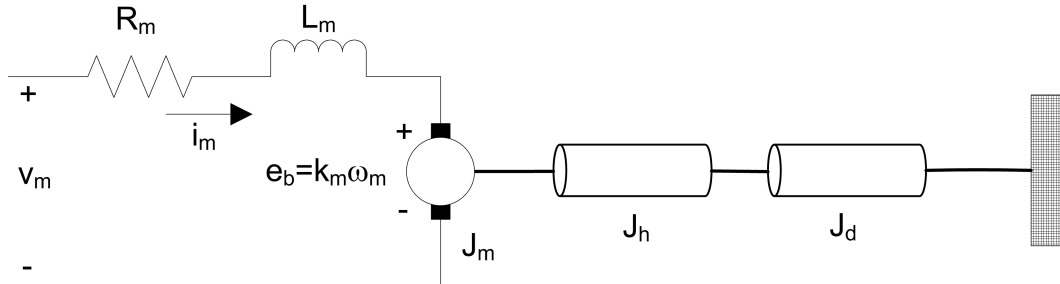


Figure 1.1: QUBE-Servo 2 DC motor and load

The back-emf (electromotive) voltage  $e_b(t)$  depends on the speed of the motor shaft,  $\omega_m$ , and the back-emf constant of the motor,  $k_m$ . It opposes the current flow. The back emf voltage is given by:

$$e_b(t) = k_m \omega_m(t) \quad (1.1)$$

Symbol	Description	Value
<b>DC Motor</b>		
$R_m$	Terminal resistance	$8.4\Omega$
$k_t$	Torque constant	$0.042 \text{ N.m/A}$
$k_m$	Motor back-emf constant	$0.042 \text{ V/(rad/s)}$
$J_m$	Rotor inertia	$4.0 \times 10^{-6} \text{ kg.m}^2$
$L_m$	Rotor inductance	$1.16 \text{ mH}$
$m_h$	Load hub mass	$0.0106 \text{ kg}$
$r_h$	Load hub mass	$0.0111 \text{ m}$
$J_h$	Load hub inertia	$0.6 \times 10^{-6} \text{ kg.m}^2$
<b>Load Disk</b>		
$m_d$	Mass of disk load	$0.053 \text{ kg}$
$r_d$	Radius of disk load	$0.0248 \text{ m}$

Table 1.1: QUBE-Servo 2 system parameters

Using Kirchoff's Voltage Law, we can write the following equation:

$$v_m(t) - R_m i_m(t) - L_m \frac{di_m(t)}{dt} - k_m \omega_m(t) = 0. \quad (1.2)$$

Since the motor inductance  $L_m$  is much less than its resistance, it can be ignored. Then, the equation becomes

$$v_m(t) - R_m i_m(t) - k_m \omega_m(t) = 0. \quad (1.3)$$

The motor shaft equation is expressed as

$$J_{eq}\dot{\omega}_m(t) = \tau_m(t), \quad (1.4)$$

where  $J_{eq}$  is total moment of inertia acting on the motor shaft and  $\tau_m$  is the applied torque from the DC motor. Based on the current applied, the torque is

$$\tau_m = k_t i_m(t) \quad (1.5)$$

The moment of inertia of a disk about its pivot, with mass  $m$  and radius  $r$ , is

$$J = \frac{1}{2}mr^2. \quad (1.6)$$

## 2 In-Lab Exercises

Based on the QUARC controller designed in QUBE-Servo 2 Filtering laboratory experiment, design a controller that can apply a constant voltage to the motor and read the servo velocity using the encoder (or the tachometer on channel 14000) as shown in Figure 2.1. Configure the HIL Read block to also read from the current sensor on analog input channel #0.

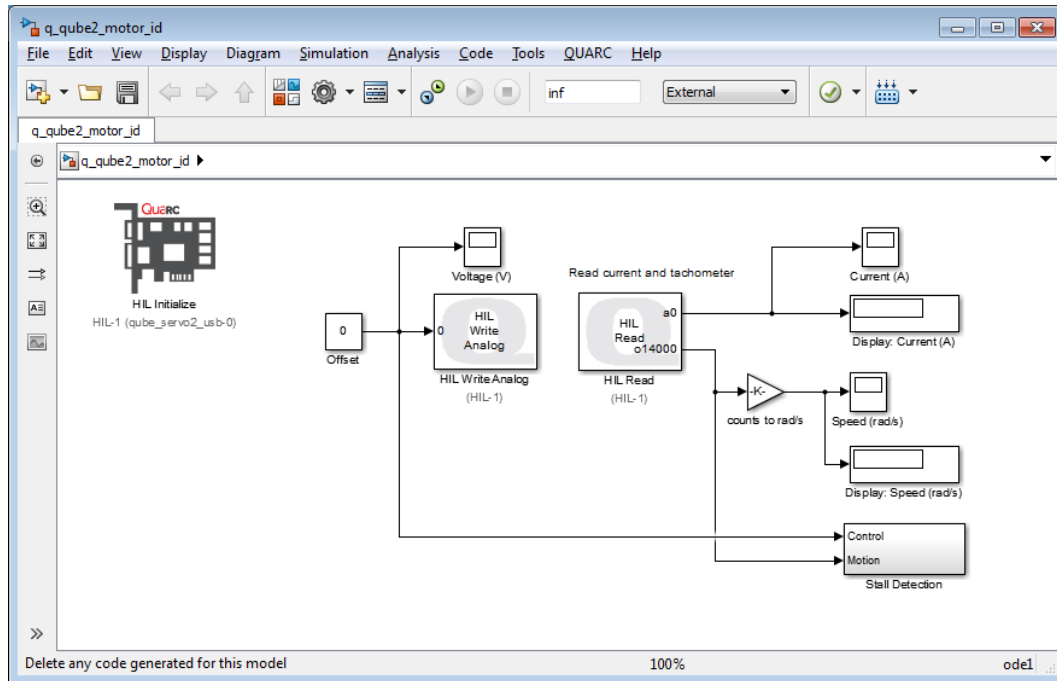


Figure 2.1: QUARC controller that applies a voltage to the DC motor and measures motor current and speed

1. During the in-laboratory session you will be experimentally estimating the motor resistance  $R_m$ . This can be done by applying constant voltages to the motor and measuring the corresponding current *while holding the motor shaft stationary*. Derive an expression that will allow you to solve for  $R_m$  under these conditions.
2. During the in-laboratory session you will be experimentally estimating the motor torque constant  $k_m$ . This can be done by applying constant voltages to the motor and measuring both corresponding steady-state current and speed (in radians per second). Assuming that the motor resistance is known, derive an expression that will allow you to solve for  $k_m$ .
3. Connect and power up the QUBE-Servo 2 system and ensure the inertial disc load is mounted.
4. To experimentally estimate the motor resistance, apply a set of voltages to the QUBE-Servo 2 using the model in Figure 2.1. For each measurement, *hold the motor shaft stationary* by grasping the inertial disc load to stall the motor and record the current measurement displayed in the *Current (A)* display. Fill the following table with the measured current for different voltage and calculate the corresponding resistance.

Applied Voltage: $V_m$ (V)	Measured Current: $I_m$ (A)	Resistance: $R_m$ ( $\Omega$ )
-5		
-4		
-3		
-2		
-1		
1		
2		
3		
4		
5		

Table 2.1: Motor resistance experimental results

- Take the average of all the measured resistance values and compare this with the motor resistance value from the specification sheet in Table 1.1.
- To experimentally estimate the motor back-EMF constant, repeat the same procedure by applying different voltage to the QUBE-Servo 2 with the motor free to spin (i.e. do not stall the motor) and record the measured speed and current in Table 2.2.

$V_m$ (V)	Measured Speed: $\omega_m$ (rad/s)	Measured Current: $I_m$ (A)	Back-emf: $k_m$ (V-s/rad)
-5			
-4			
-3			
-2			
-1			
1			
2			
3			
4			
5			

Table 2.2: Back-emf experimental results

- Take the average of the measured back-emf values and compare this with the motor back-emf value from the motor specification sheet given in Table 1.1.
- The motor shaft of the QUBE-Servo 2 is attached to a *load hub* and a disk load. Based on the parameters given in Table 1.1, calculate the equivalent moment of inertia that is acting on the motor shaft.
- Formulate the differential equation for  $\omega_m$  using Equation 1.2 to Equation 1.5. Compare your result with the transfer function obtained from the Step Response Modeling laboratory experiment. (**Hint:** Obtain the Voltage  $V_m(s)$  to Speed  $\Omega_m(s)$  transfer function by applying a Laplace Transform to the derived differential equation.)
- Stop QUARC and turn off the QUBE-Servo 2 if no more experiments will be conducted.

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