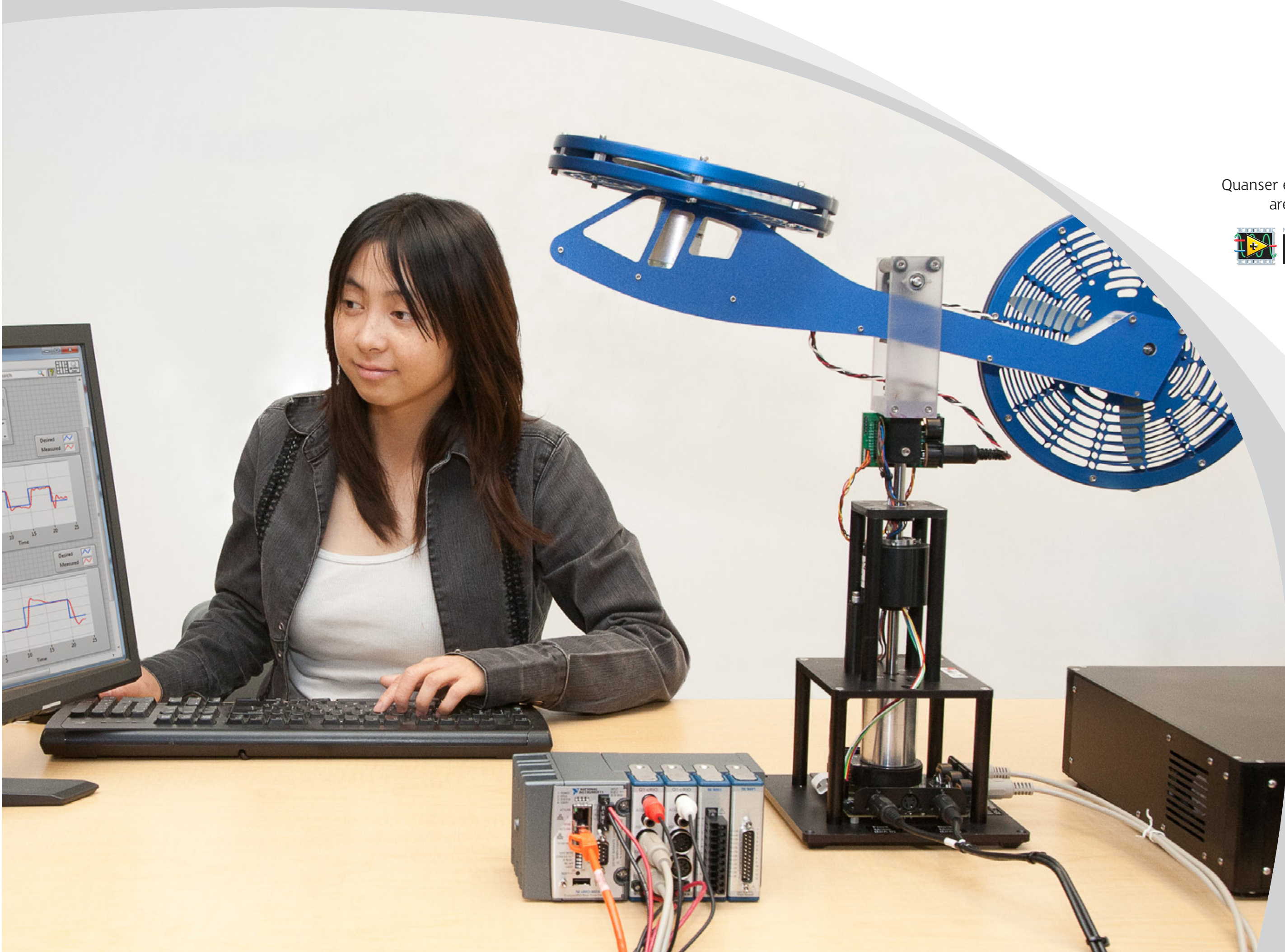
LABORATORY GUIDE

2 DOF Helicopter Experiment for LabVIEW™ Users

Developed by:
Jacob Apkarian, Ph.D., Quanser
Michel Lévis, M.A.SC., Quanser
Cameron Fulford, M.A.SC., Quanser



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Quanser Inc.
119 Spy Court
Markham, Ontario
L3R 5H6
Canada
info@quanser.com
Phone: 1-905-940-3575
Fax: 1-905-940-3576

Printed in Markham, Ontario.

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1 PRESENTATION

1.1 Description

The Quanser 2 DOF Helicopter experiment, shown in Figure 1.1, consists of a helicopter model mounted on a fixed base with two propellers that are driven by DC motors. The front propeller controls the elevation of the helicopter nose about the pitch axis and the back propeller controls the side to side motions of the helicopter about the yaw axis. The pitch and yaw angles are measured using high-resolution encoders. The pitch encoder and motor signals are transmitted via a slipring. This eliminates the possibility of wires tangling on the yaw axis and allows the yaw angle to rotate freely about 360 degrees.

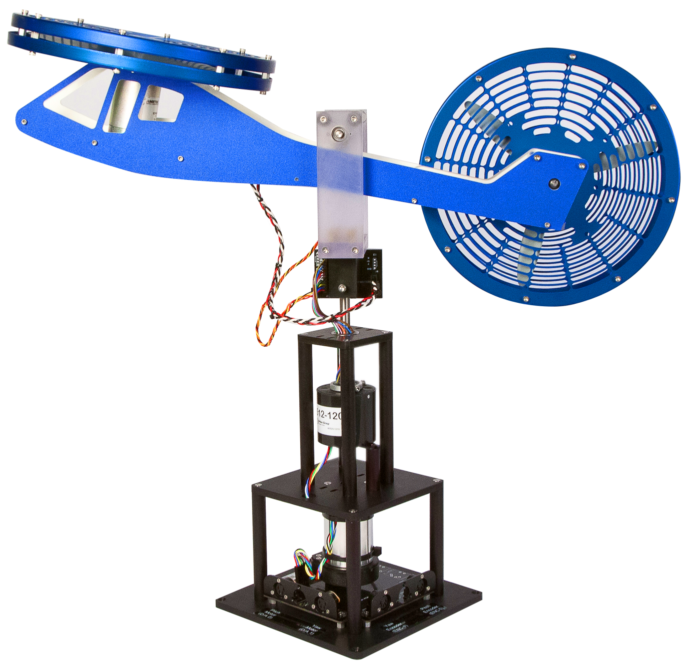


Figure 1.1: Quanser 2 DOF Helicopter

The modeling and position control design of the helicopter are summarized in section 3. In section 5, several procedures are outlined that show how to simulate the position controller and how to run this controller on the actual helicopter plant. Further, this section explains how to use the joystick to manually control the helicopter.

1.2 Prerequisites

In order to successfully carry out this laboratory, the user should be familiar with the following:

- 2 DOF Helicopter main components (e.g. actuator, sensors), the data acquisition card (e.g., Q2-USB), and the power amplifier (e.g. VoltPaq), as described in [3], [1], and [2], respectively.
- Wiring the 2 DOF Helicopter plant with the amplifier and data-acquisition device, as discussed in the 2 DOF Helicopter User Manual ([3]).
- Designing a state-feedback control using Linear-Quadratic Regulator (LQR).
- Using LabVIEW™ to design, control, and monitor a plant in real-time.

2 EXPERIMENT FILES OVERVIEW

Table 2.1 below lists and describes the various files supplied with the 2 DOF Helicopter experiment.

File Name	Description
2 DOF Helicopter User Manual.pdf	This manual is the user guide for the Quanser 2 DOF Helicopter specialty aerospace plant. It contains information about the hardware components, specifications, and information to setup and configure the hardware.
2 DOF Helicopter Laboratory Guide.pdf	This manual is the laboratory guide for the Quanser 2 DOF Helicopter specialty aerospace plant. It contains information about the system modeling, control design, as well as the experimental procedure to simulate and implement the controller.
2DOF Helicopter Equations.mws	Maple worksheet used to analytically derive the state-space model involved in the experiment. Waterloo Maple 9, or a later release, is required to open, modify, and execute this file.
2DOF Helicopter Equations.html	HTML presentation of the Maple Worksheet. It allows users to view the content of the Maple file without having Maple 9 installed. No modifications to the equations can be performed when in this format.
2D HELI LAB.lvproj	LabVIEW project file that contains all of the LabVIEW files needed to run the 2 DOF Helicopter experiment.
2D HELI Modeling.vi	Used to generate a state-space model for the 2 DOF Helicopter .
2D HELI Control Design.vi	Used with the model generated in the *2D HELI Modeling.vi* to design a state feedback controller for the 2 DOF Helicopter .
2D HELI Control Simulation.vi	Used with the model and state feedback controller gains to simulate the 2 DOF Helicopter response.
2 DOF HELI Control.vi	Uses the Quanser Rapid Control Prototyping (RCP) Toolkit to control the 2 DOF Helicopter with a joystick input device.
2 DOF HELI Control (cRIO).vi	Uses the Quanser Rapid Control Prototyping (RCP) Toolkit to control the 2 DOF Helicopter for users of the National Instruments cRIO. Used with *Joystick_Read.vi* to allow joystick control of the 2 DOF Helicopter .
Joystick_Read.vi	Acts as a joystick host VI when the 2 DOF Helicopter is connected to the National Instruments cRIO and the user wishes to command the vehicle with a joystick. Joystick data is transmitted from the desktop PC to the cRIO using the RCP stream functionality.
Joystick Test.vi	Used to test the USB joystick.

Table 2.1: Files supplied with the 2 DOF Helicopter experiment

3 MODELING

3.1 Dynamics

The free-body diagram of the 2 DOF Helicopter is illustrated in Figure 3.1 and it accompanies the Maple worksheet named *2DOF Helicopter Equations.mws* or its HTML equivalent *2DOF Helicopter Equations.html*. The equations can be edited and re-calculated by executing the worksheet using Maple 9.

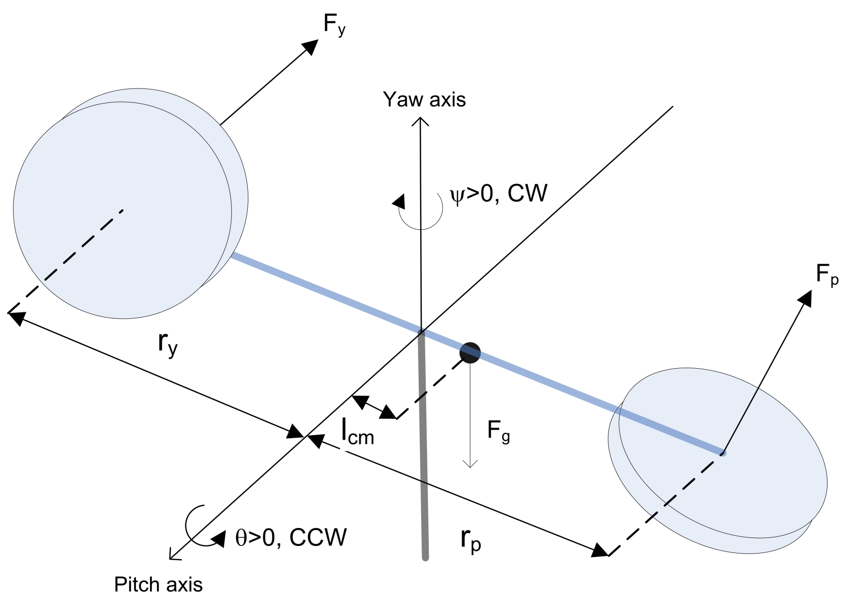


Figure 3.1: Simple free-body diagram of 2 DOF Helicopter

The 2 DOF Helicopter modeling conventions used are:

1. The helicopter is horizontal when the pitch angle equals $\theta = 0$.
2. The pitch angle increases positively, $\dot{\theta(t)} > 0$, when the nose is moved upwards and the body rotates in the **counter-clockwise (CCW)** direction.
3. The yaw angle increases positively, $\dot{\psi(t)} > 0$ when the body rotates in the **clockwise (CW)** direction.
4. Pitch increases, θ>0, when the pitch thrust force is positive $F_p > 0$.
5. Yaw increases, ψ>0, when the yaw thrust force is positive, $F_y > 0$.

The Maple worksheet goes through the kinematics and dynamics of the system. The Cartesian coordinates of the center-of-mass are expressed relative to the base coordinate system, as shown in Figure 3.2. These resulting equations are used to find the potential energy and translational kinetic energy.

3.2 State-Space Model

The thrust forces acting on the pitch and yaw axes from the front and back motors are then defined. Using the Euler-Lagrange formula, the nonlinear equations of motion of the 2 DOF Helicopter system are derived. These equations are linearized about zero and the linear state-space model (A,B,C,D) describing the voltage-to-angular joint position dynamics of the system is found. Given the state-space representation

$$\dot{x} = Ax + Bu$$

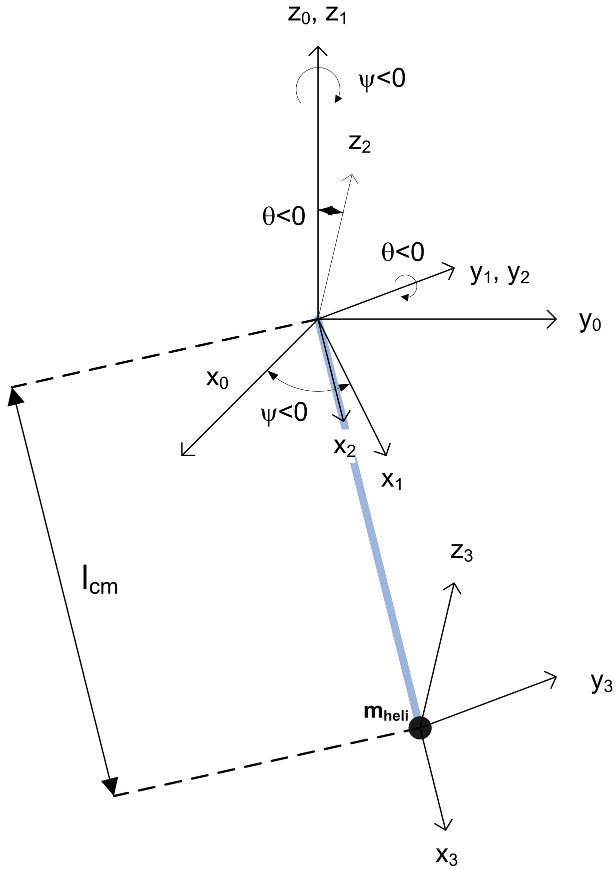


Figure 3.2: Kinematics of the 2 DOF Helicopter

and

$$y = Cx + Du$$

the state vector for the 2 DOF Helicopter is defined

$$x^T = \left[\theta(t),\ \psi(t),\ \dot{\theta(t)},\ \dot{\psi(t)}\right] \tag{3.1}$$

and the output vector is

$$y^T = [\theta(t),\ \psi(t)]$$

where θ and ψ are the pitch and yaw angles, respectively. The corresponding helicopter state-space matrices (as derived in the Maple worksheet) are

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{B_p}{J_{Tp}} & 0 \\ 0 & 0 & 0 & -\frac{B_y}{J_{Ty}} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{K_{pp}}{J_{Tp}} & \frac{K_{py}}{J_{Tp}} \\ \frac{K_{yp}}{J_{Ty}} & \frac{K_{yy}}{J_{Ty}} \end{bmatrix}$$

where

$$\begin{aligned} J_{Tp} &= J_{eq\_p} + m_{heli} l_{cm}^2 \\ J_{Ty} &= J_{eq\_y} + m_{heli} l_{cm}^2 \end{aligned}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The model parameters used in the (A,B) matrices are defined in the 2 DOF Helicopter User Manual.

4 CONTROL DESIGN

4.1 State-Feedback

In this section a state-feedback controller is designed to regulate the elevation and travel angles of the 2 DOF Helicopter to desired positions. However, as will be shown, the control structure is basically linear proportional-integral-derivative, i.e. PID, controller. The control gains are computed using the Linear-Quadratic Regulator algorithm in Section 4.2.

The state-feedback controller entering the front motor, u_f, and the back motor, u_b, is defined

$$\begin{bmatrix} u_p \\ u_y \end{bmatrix} = K_{PD}(x_d - x) + V_i + \begin{bmatrix} u_{ff} \\ 0 \end{bmatrix},$$

with the proportional-derivative gain

$$K_{PD} = \begin{bmatrix} k_{1,1} & k_{1,2} & k_{1,3} & k_{1,4} \\ k_{2,1} & k_{2,2} & k_{2,3} & k_{2,4} \end{bmatrix},$$

the desired state

$${x_d}^T = \begin{bmatrix} \theta_d & \psi_d & 0 & 0 \end{bmatrix},$$

the integral control

$$V_i = \begin{bmatrix} k_{1,5} \int (x_{d,1} - x_1)dt + k_{1,6} \int (x_{d,2} - x_2)dt \\ k_{2,5} \int (x_{d,1} - x_1)dt + k_{2,6} \int (x_{d,2} - x_2)dt \end{bmatrix},$$

and the nonlinear feed-forward control

$$u_{ff} = \frac{K_{ff} m_{heli} g l_{cm} \cos x_{d,1}}{K_{pp}}.$$

The feed-forward control compensates for the gravitational torque that forces the pitch angle down. The system state, x, is defined in Equation 3.1. The variables θ_d and λ_d, are the pitch and yaw setpoints, i.e. the desired angles of the helicopter. In state-space, the desired pitch is angle $x_{d,1}$ and the desired yaw is $x_{d,2}$. The gains $k_{1,1}$ and $k_{1,2}$ are the front motor control proportional gains and the gains $k_{2,1}$ and $k_{2,2}$ are the back motor control proportional gains. Next, $k_{1,3}$ and $k_{1,4}$ are the front motor control derivative gains and $k_{2,3}$ and $k_{2,4}$ are the back motor control derivative gains. The integral control gains used in the front motor control are $k_{1,5}$ and $k_{1,6}$ and the integral gains $k_{2,5}$ and $k_{2,6}$ are used in the back motor regulator.

4.2 Linear Quadratic Regulator

The control gains are computed using the Linear-Quadratic Regulator scheme. The system state is first augmented to include the integrals of the pitch and yaw states,

$${x_i}^T = \begin{bmatrix} \theta & \psi & \dot{\theta} & \dot{\psi} & \int \theta dt & \int \lambda dt \end{bmatrix}$$

Using the feedback law

$$u = -K\, x_i$$

the weighting matrices

$$Q = \begin{bmatrix} 200 & 0 & 0 & 0 & 0 & 0 \\ 0 & 150 & 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 & 0 \\ 0 & 0 & 0 & 200 & 0 & 0 \\ 0 & 0 & 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 0 & 0 & 50 \end{bmatrix}$$

and

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and the state-space matrices (A,B) found previously, the control gain

$$K = \begin{bmatrix} 18.9 & 1.98 & 7.48 & 1.53 & 7.03 & 0.770 \\ -2.22 & 19.4 & -0.45 & 11.9 & -0.770 & 7.03 \end{bmatrix}$$

is calculated by minimizing the cost function

$$J = \int_0^\infty x_i{}^T \, Q \, x_i \, + \, u^T \, R \, u \, dt.$$

$$K = \begin{bmatrix} k_{1,1} & k_{1,2} & k_{1,3} & k_{1,4} & k_{1,5} & k_{1,6} \\ k_{2,1} & k_{2,2} & k_{2,3} & k_{2,4} & k_{2,5} & k_{2,6} \end{bmatrix}$$

4.3 Anti-Windup

The helicopter system runs the risk of integrator windup. That is, given a large error in the between the measured and desired pitch angle, $\theta - \theta_d$, or between the measured and desired yaw angle, $\psi - \psi_d$, the integrator outputs a large voltage that can saturate the amplifier. By the time the measured angle reaches the desired angle the integrator built-up so much energy that it remains saturated. This can cause large overshoots and oscillations in the response. To fix this, an integral windup protection algorithm is used. Figure 4.1 illustrates the anti-windup scheme implemented to control the pitch.

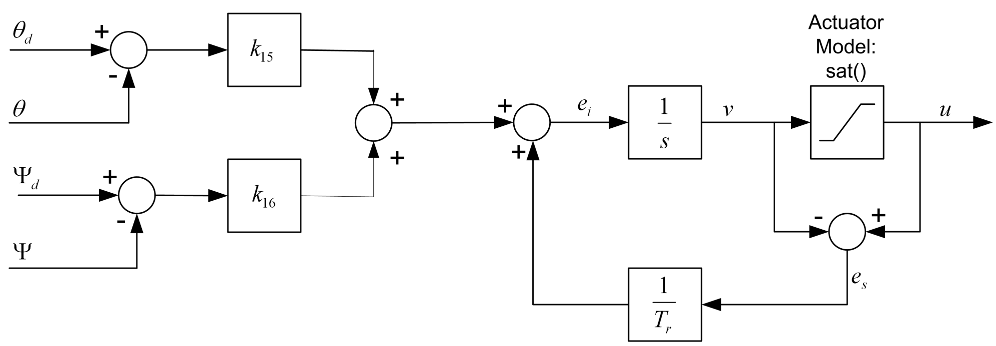


Figure 4.1: Anti-windup loop

The integrator input shown in the windup loop is

$$e_i = k_{1,5} \, (\theta_d - \theta) \, + \, k_{1,6} \, (\psi_d - \psi) \, + \, \frac{u \, - \, v}{T_r}$$

When the integrator output voltage, v, is larger than the imposed integral saturation then the saturation error becomes negative, $e_s < 0$. The saturation error gets divided by the reset time, T_r, and its result is added to the integrator input. This effectively decreases the integrator input and winds-down the integrator. In the simulation and experimental results the saturation limit of the integrator is set to 5 V and the reset time to 1 sec for maximum wind-down speed.

5 IN-LAB PROCEDURE

5.1 2 DOF Helicopter LabVIEW Files

The LabVIEW files supplied with the 2 DOF Helicopter contain various controls that implement the model and controllers presented previously. The *2 DOF HELI Control* VI implements the feed-forward control and the LQR PID position controller discussed in section 4.

Note: For National Instruments cRIO users, the *2 DOF HELI Control (cRIO)* VI must be used instead of *2 DOF HELI Control*.

The block diagram of the *2 DOF HELI Control* VI is displayed in Figure 5.1. As discussed in Section 4, the position and velocity states are multiplied by the corresponding elements of control gain K. The state includes the integral of the pitch and yaw angles and those are multiplied by the integral gains in K.

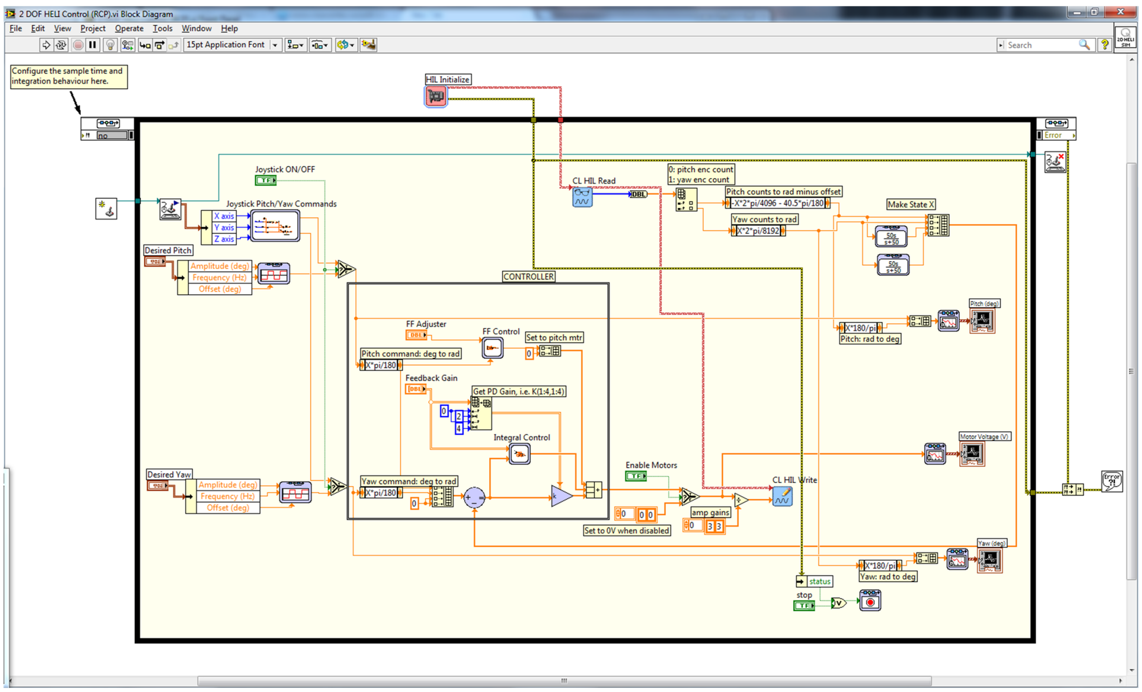


Figure 5.1: Block diagram that implements the FF+LQR+I controller

5.2 Modeling, Control Design and Simulation

The following sections describe how to utilize LabVIEW™ and the Quanser Rapid Control Prototyping Toolkit® to develop the model, closed-loop controller, and simulation of the 2 DOF Helicopter experiment.

5.2.1 Objectives

- Generate a linear state-space model from the 2 DOF Helicopter system parameters
- Save the state-space model to a file that will be used for LQR control
- Design a LQR feedback controller to stabilize the 2 DOF Helicopter plant
- Simulate the performance of the LQR controller using the 2 DOF Helicopter nonlinear model

5.2.2 Modeling Procedure

Follow these steps to generate the state-space model of the 2 DOF Helicopter:

1. Load the LabVIEW™ software.
2. Open the LabVIEW project called *2D HELI LAB.lvproj*, shown in Figure 5.2.

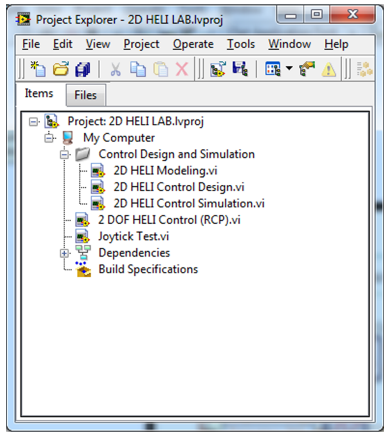


Figure 5.2: LabVIEW project used for the 2 DOF Helicopter system

3. Under the *Control Design and Simulation* directory in the project explorer, open the *2D HELI Modeling* VI.
4. The front panel of the *2D HELI Modeling* VI is shown in Figure 5.3.

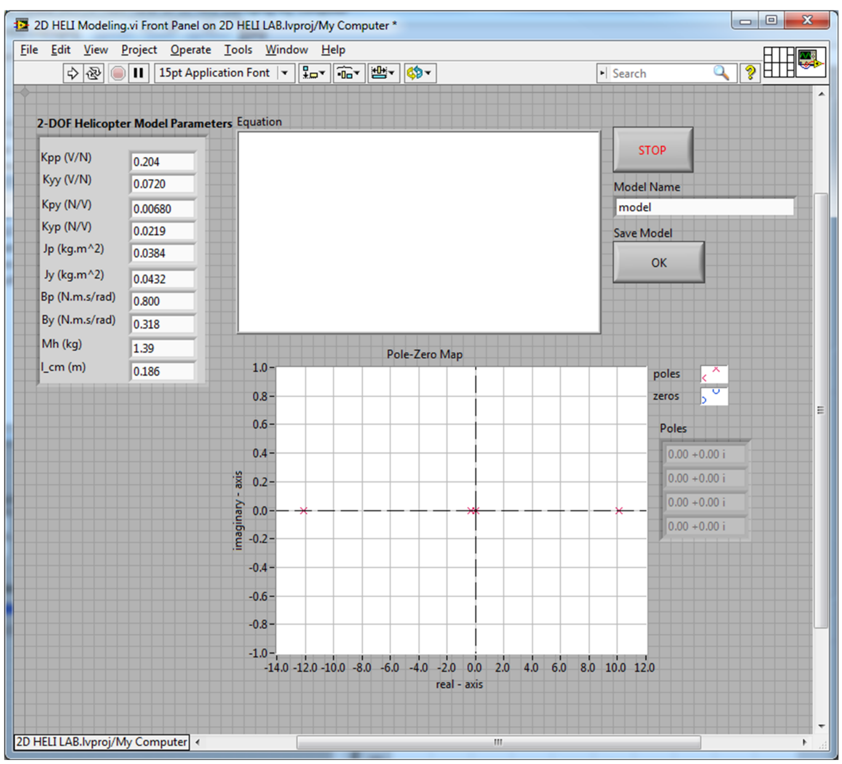


Figure 5.3: Model of the 2 DOF Helicopter

5. Ensure all of the 2 DOF Helicopter model parameters are set in the VI list of parameters.

6. Run the VI. The resulting state-space model is shown in the Equation display on the front panel, as shown in Figure 5.4.

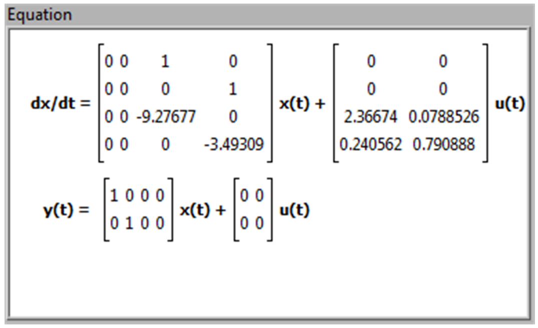


Figure 5.4: State-space model equations for the 2 DOF Helicopter

7. While the VI is running, click the OK button on the front panel to save the model to a file. This model file is used throughout the rest of the control design and simulation VIs.

5.2.3 Controller Design Procedure

Follow these steps to design a LQR controller for the 2 DOF Helicopter linear model:

1. From the project explorer, open the *2D HELI Control Design* VI.
2. Run the *2D HELI Control Design* VI and when prompted select the model file saved from the *2D HELI Modeling* VI.
3. Verify that the Controllability tab shows that the system has 4 states and that the controllability matrix rank is 4 and thus controllable, as shown in Figure 5.5.

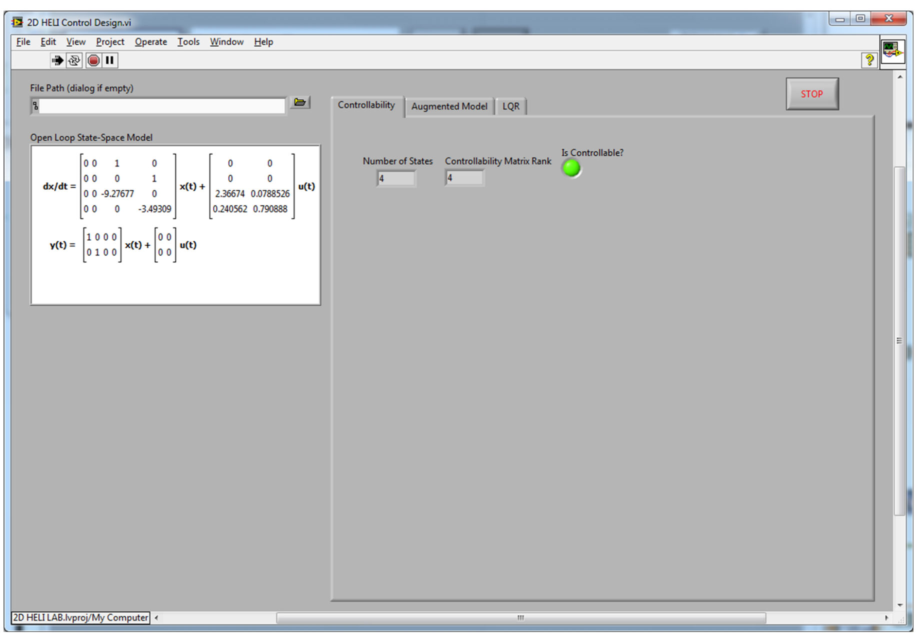


Figure 5.5: State-space model controllability

4. Click on the Augmented Model tab to generate the linear 2 DOF Helicopter model augmented with integrated states $\int \theta dt$ and $\int \lambda dt$, as shown in Figure 5.6.

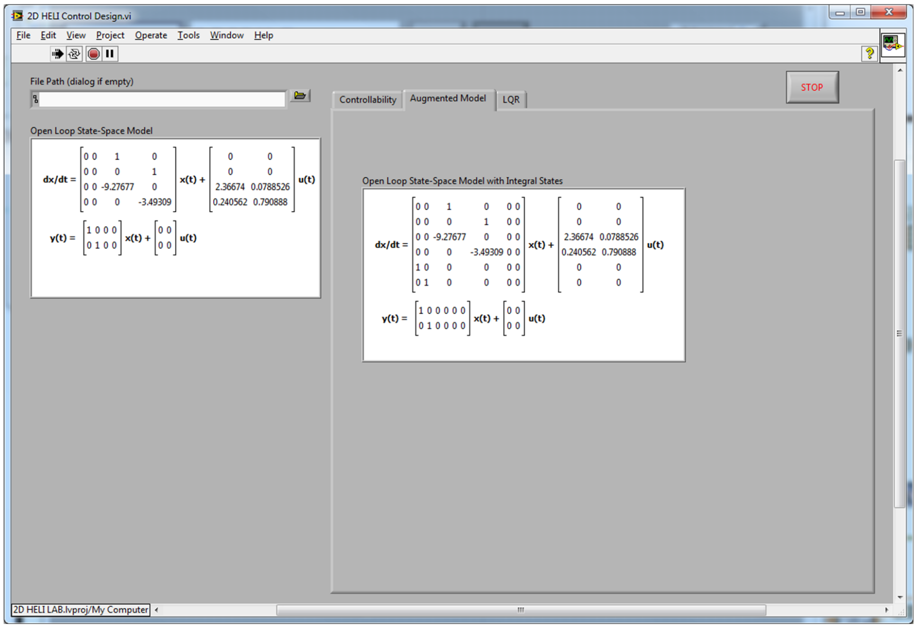


Figure 5.6: Linear model augmented with integrated states.

5. Click on the LQR tab and verify that the generated feedback gain K will stabilize the system, as shown in Figure 5.7. You can change the Q and R matrices to adjust the weighting parameters of the LQR controller design.

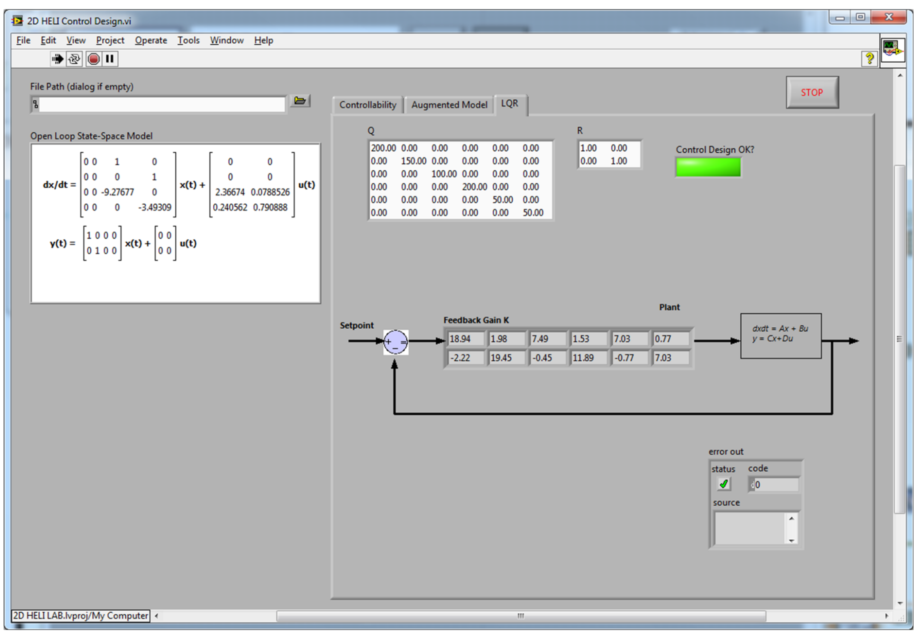


Figure 5.7: LQR controller design

6. Record the LQR feedback gain K for use in the simulation and implementation of the controller in the next sections. You can return to this VI to regenerate new feedback gains after testing the performance of the closed-loop controller.

5.2.4 Closed-Loop Simulation Procedure

Follow these steps to simulate the performance of the closed-loop LQR controller with the 2 DOF Helicopter nonlinear model:

1. Open the *2D HELI Control Simulation* VI (shown in Figure 5.8), which is used to simulate the closed-loop controller with the full 2 DOF Helicopter nonlinear model.

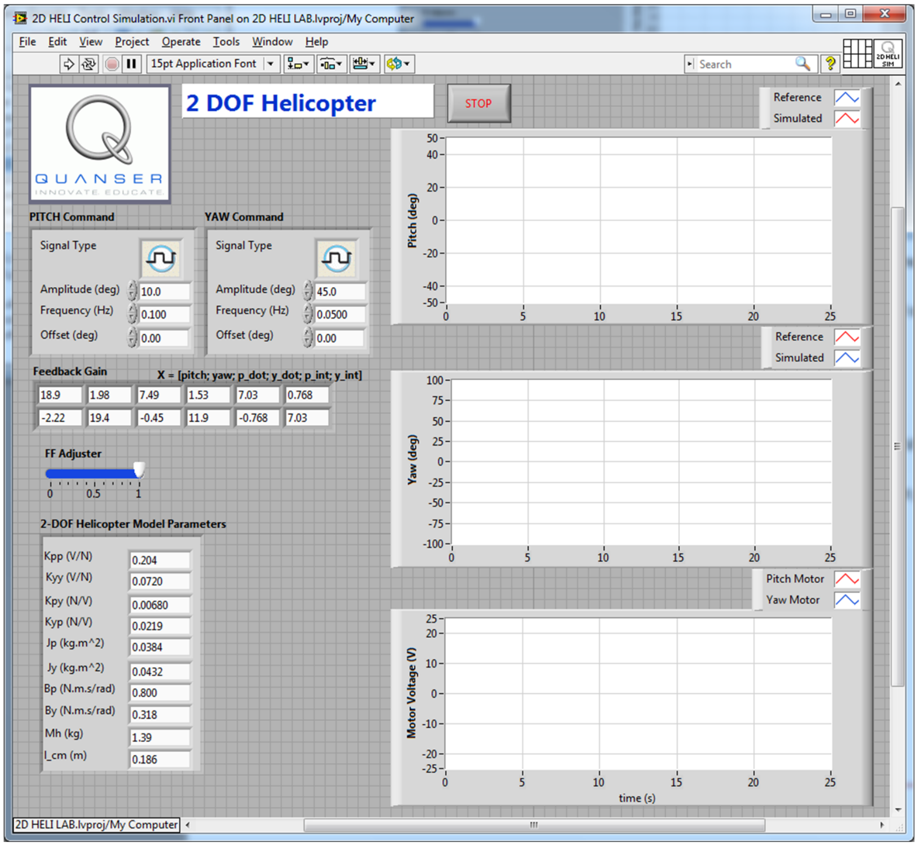


Figure 5.8: Simulation of the feedback controller using the 2 DOF Helicopter nonlinear model.

2. Input the LQR feedback gain K into the front panel of the *2D HELI Control Simulation* VI shown in Figure 5.9.

Feedback Gain — X = [pitch; yaw; p_dot; y_dot; p_int; y_int]

18.9	1.98	7.49	1.53	7.03	0.768
-2.22	19.4	-0.45	11.9	-0.768	7.03

Figure 5.9: Input the feedback gain into the *2D HELI Control Simulation* VI

3. Check that the 2 DOF Helicopter model parameters are entered correctly on the VI front panel.
4. Run the *2D HELI Control Simulation* VI.
5. Observe the closed-loop simulation response as shown in Figure 5.10.

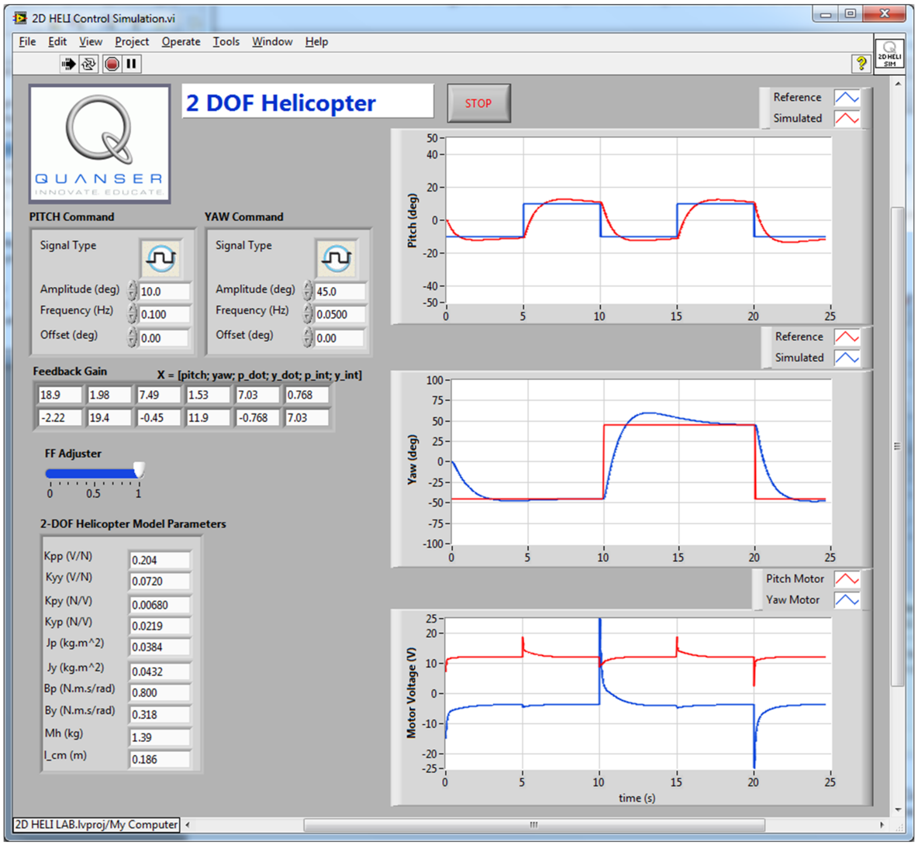


Figure 5.10: Simulation response for the 2 DOF Helicopter

6. The command set points for the pitch and yaw angles can be changed with the PITCH Command and YAW Command front panel controls. Adjust the commands and observe the tracking performance of the simulated response.

5.3 Closed-loop Position Control Implementation

5.3.1 Objectives

The objectives of running the 2 DOF Helicopter in closed-loop are to:

- Investigate the closed-loop performance of the FF+LQR+I controller running on the actual 2 DOF Helicopter plant.

5.3.2 Procedure: 2 DOF Helicopter

Follow this procedure to run the FF+LQR+I controller on the actual helicopter plant:

1. Load LabVIEW™.
2. Open the LabVIEW project called *2D HELI LAB.lvproj*.
3. Open the *2D HELI Control* VI that is used to run the 2 DOF Helicopter experiment, shown in Figure 5.11.

   Note: National Instruments cRIO users should open the *2D HELI Control (cRIO)* VI instead of *2D HELI Control*.

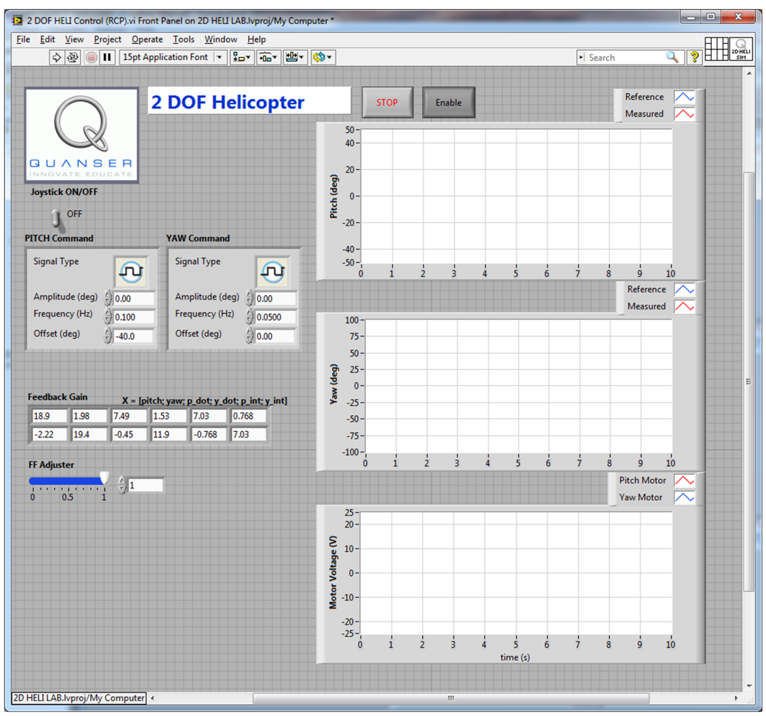


Figure 5.11: LabVIEW VI used to run the closed-loop controller on the 2 DOF Helicopter

4. Using the feedback gain K generated from Section 5.2.2, input the feedback gain K into the front panel of the VI.
5. Open the *2D HELI Control* VI (or the *2D HELI Control (cRIO)* VI for cRIO users) block diagram (shortcut CTRL+E) shown in Figure 5.12. It contains the RCP controls that interface with the hardware of the actual plant. The Sim HIL Write Analog control outputs the voltage computed by the controller to the DAQ board and the Sim HIL Read Encoder control reads the encoder measurements.

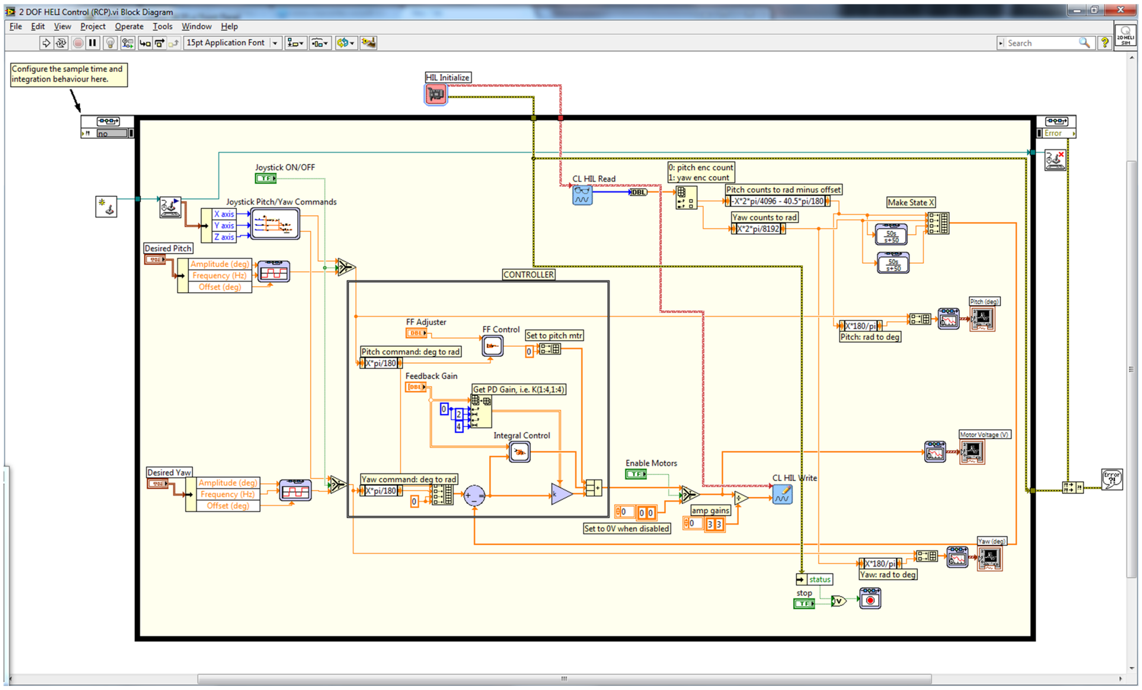


Figure 5.12: Block diagram used to run the closed-loop controller on the 2 DOF Helicopter

6. **Configure DAQ**: Double-click on the HIL Initialize control and ensure it is configured for the DAQ device that is installed in your system. In this VI, the default is setup for the Quanser Q2-USB hardware-in-the-loop board.
7. Ensure the helicopter has been setup and all the connections have been made as instructed in the 2 DOF Helicopter user manual.
8. Turn ON the amplifier. For the VoltPAQ-X2, the green LED on the amplifier should be lit.
9. In the VI front panel make sure the Joystick ON/OFF switch is set to the OFF position.
10. Run the *2D HELI Control* VI (or the *2D HELI Control (cRIO)* VI for cRIO users). You should now hear the propellers running.
11. In the *PITCH Command* section, slowly ramp up the *Offset* value to 0 degrees to bring the helicopter up to horizontal.
12. Set the *PITCH Command Amplitude* to 10 degrees and see the response in Figure 5.13, which depicts the typical measured response under a step pitch angle. The measured response is the red line and the reference is the blue line.

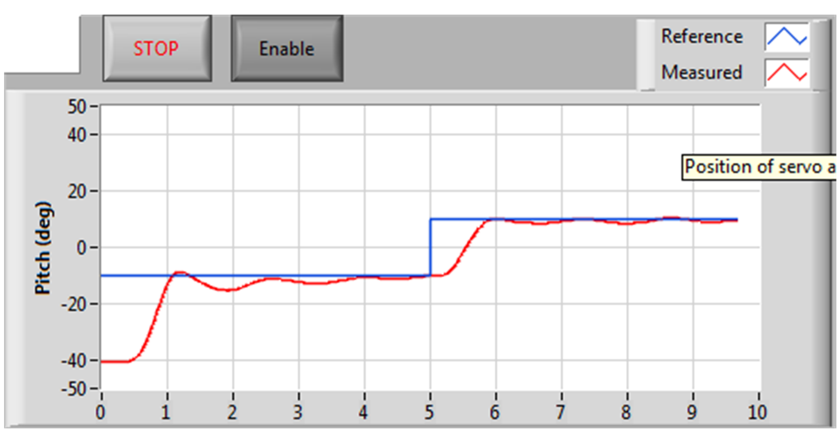


Figure 5.13: Closed-loop LQR response under pitch reference step

13. Inside the *PITCH Command* front panel control set the *Amplitude (deg)* to 0 and inside the *YAW Command* front panel control set the *Amplitude (deg)* to 20. The helicopter should track the commanded yaw angle.

14. Figure 5.14 depicts the typical measured yaw response given a desired step yaw angle.

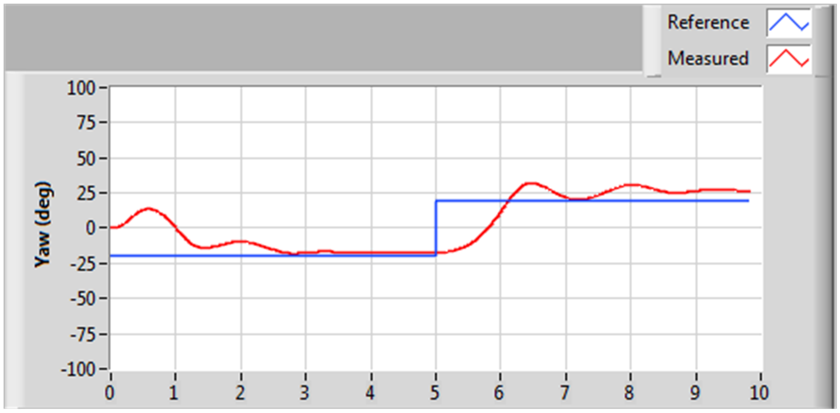


Figure 5.14: Closed-loop LQR response under yaw reference step

15. Alternatively, the desired angle can be generated using the joystick. To use the joystick, set the *Joystick ON/OFF* switch shown in Figure 5.11 to the ON position. The rate at which the desired angle increases or decreases given a joystick position can be changed using the *Rate Command* knob. When starting, set the *Rate Command* knob on the joystick to the midpoint position.

    Note: For users of the National Instruments cRIO, the *Joystick_Read* VI must be used in conjunction with the *2D HELI Control (cRIO)* VI. The cRIO does not provide joystick support, so to overcome this the *Joystick_Read* VI is executed on the desktop PC. Upon opening the *Joystick_Read* VI, the user must enter the IP address of the cRIO in the *cRIO IP Address* field of the front panel. The VI must then be started prior to starting *2D HELI Control (cRIO)*.

    ■ **Caution: Do not switch the joystick ON when the controller is running**. Set the joystick switch before starting the VI if the joystick is to be used.

16. Gradually bring the helicopter back to starting position.

17. Click on the *Stop* button on the VI front panel to stop running the system.

18. Power off the amplifier.

6 TECHNICAL SUPPORT

To obtain support from Quanser, go to http://www.quanser.com/ and click on the Tech Support link. Fill in the form with all the requested software and hardware information as well as a description of the problem encountered. Also, make sure your e-mail address and telephone number are included. Submit the form and a technical support representative will contact you.

REFERENCES

[1] Quanser Inc. *Q2-USB Data-Acquisition System User's Guide*, 2010.

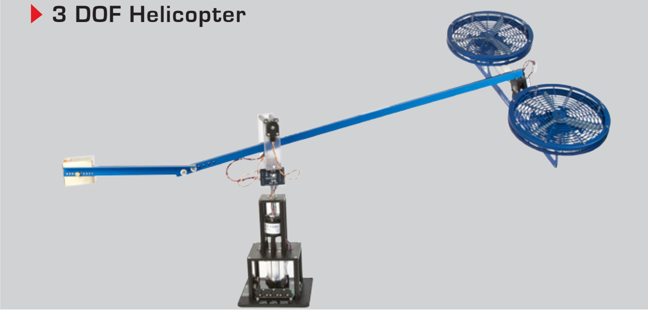
[2] Quanser Inc. *VoltPAQ User Guide*, 2010.

[3] Quanser Inc. *2D Helicopter User Manual*, 2011.

Aerospace plants for teaching and research

2 DOF Helicopter

3 DOF Helicopter



These plants are ideal for intermediate or advanced level teaching. They are also suitable for research relating to traditional or modern control applications of aerospace engineering. For more information please contact info@quanser.com.

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