Half-Quad PD Control Design

Note: This requires the *Symbolic Math Toolbox*.

```
clear;
syms Kt Dt Jy Dy Kf s;
syms y yd u kp kd;
syms wn zeta;
plant_tf = y == Kf*Dt/Jy / (s^2 + Dy/Jy*s ) * u;
control_tf = u == kp*(yd-y) - kd*s*y;
tf_eqns = [plant_tf,control_tf]
```

```
tf_eqns =
```

$$\left(y = \frac{\mathrm{Dt}\ \mathrm{Kf}\ u}{\mathrm{Jy}\ \left(s^2 + \frac{\mathrm{Dy}\ s}{\mathrm{Jy}}\right)} \quad u = -\mathrm{kp}\ (y - \mathrm{yd}) - \mathrm{kd}\ s\ y \right)$$

```
S = solve(tf_eqns,[y u]);
% Display symbolic solution returned by solve
display(S.y);
```

```
ans =
```

$$\frac{\mathrm{Dt}\ \mathrm{Kf}\ \mathrm{kp}\ \mathrm{yd}}{\mathrm{Dy}\ s + \mathrm{Jy}\ s^2 + \mathrm{Dt}\ \mathrm{Kf}\ \mathrm{kp} + \mathrm{Dt}\ \mathrm{Kf}\ \mathrm{kd}\ s}$$

```
% prototype 2nd order equation
prototype_eqn = s^2+2*zeta*wn*s+wn^2;
% aero closed-loop transfer function denominator
aero_denom = (Dy*s + Jy*s^2 + Dt*Kf*kp + Dt*Kf*kd*s)/Jy;
% find coefficicients
aero_denom_coeff = coeffs(aero_denom,s)
```

```
aero_denom_coeff =
```

$$\left(\frac{\mathrm{Dt}\ \mathrm{Kf}\ \mathrm{kp}}{\mathrm{Jy}} \quad \frac{\mathrm{Dy} + \mathrm{Dt}\ \mathrm{Kf}\ \mathrm{kd}}{\mathrm{Jy}} \quad 1\right)$$

```
prototype_eqn_coeff = coeffs(prototype_eqn,s)
```

```
prototype_eqn_coeff =
```

$$\left(\mathrm{wn}^2 \quad 2\ \mathrm{wn}\ \zeta \quad 1\right)$$

```
% generate PD equations based on 3rd order design
p_eqns = aero_denom_coeff(1) == prototype_eqn_coeff(1)
```

```
p_eqns =
```

$$\frac{\mathrm{Dt}\ \mathrm{Kf}\ \mathrm{kp}}{\mathrm{Jy}} = \mathrm{wn}^2$$

```
d_eqns = aero_denom_coeff(2) == prototype_eqn_coeff(2)
```

```
d_eqns =
```

$$\frac{\mathrm{Dy} + \mathrm{Dt}\ \mathrm{Kf}\ \mathrm{kd}}{\mathrm{Jy}} = 2\ \mathrm{wn}\ \zeta$$

```
% solve for PID gains
Sp = solve(p_eqns,kp)
```

```
Sp =
```

$$\frac{\mathrm{Jy}\ \mathrm{wn}^2}{\mathrm{Dt}\ \mathrm{Kf}}$$

```
Sd = solve(d_eqns,kd)
```

```
Sd =
```

$$-\frac{\mathrm{Jy}\ \left(\frac{\mathrm{Dy}}{\mathrm{Jy}} - 2\ \mathrm{wn}\ \zeta\right)}{\mathrm{Dt}\ \mathrm{Kf}}$$