# Pitch Parameter Estimation

This lab shows how to find the thrust, damping, and stiffness parameters of the pitch-only system (i.e., yaw axis is locked).

## Concept Review

The equation of motion of the Aero 2 when condifugred as a 1 DOF pitch-only system is

$$J_p\ddot{\theta} + D_p\dot{\theta} + K_{sp}\theta = \tau_p = K_{pp}D_tV_p(t)$$

where $\theta$ is the pitch angle, $J_p$ is the equivalent moment of inertia acting about the pitch axis, $D_p$ is the viscous damping, $K_{sp}$ is the stiffness, $K_{pp}$ is the force thrust gain, $D_t$ is the distance from the pivot point to the propeller, $V_p$ is the voltage applied to the front or pitch rotor motor.

The corresponding open-loop transfer function is a second-order transfer function

$$P(s) = \frac{\Theta(s)}{V_p(s)} = \frac{\dfrac{D_tK_{pp}}{J_p}}{s^2 + \dfrac{D_p}{J_p}s + \dfrac{K_{sp}}{J_p}}$$

In this lab we identify the pitch thrust gain, $K_{pp}$, damping, $D_p$, and stiffness, $K_{sp}$.

## Lab Procedure

Apply a step voltage to the front rotor and then set it to 0V to see the free-oscillation response. Measure the damping, $D_p$, and stiffness, $K_{sp}$, from the free-oscillation response and the thrust, $K_{pp}$, from the step response.

### Setup

1. Make sure the Aero 2 has been tested as instructed in the Quick Start Guide.
2. Launch MATLAB and browse to the working directory that includes the Simulink models for this lab.
3. Configure the Aero 2 in the 2 DOF Helicopter configuration:
4. **Unlock** the pitch axis and **lock** the yaw axis.
5. Rear rotor 1 is **vertical** and front rotor 0 is **horizontal**.
6. Adjust masses on each rotor.
7. Connect the USB cable to your PC/laptop.
8. Connect the power and turn the power switch ON. The Aero base LED should be red.

### Lab 1: Get Open-Loop Response

The open-loop response can be obtained by running the `q_aero2_pitch_step` Simulink model shown Figure 1 with QUARC.

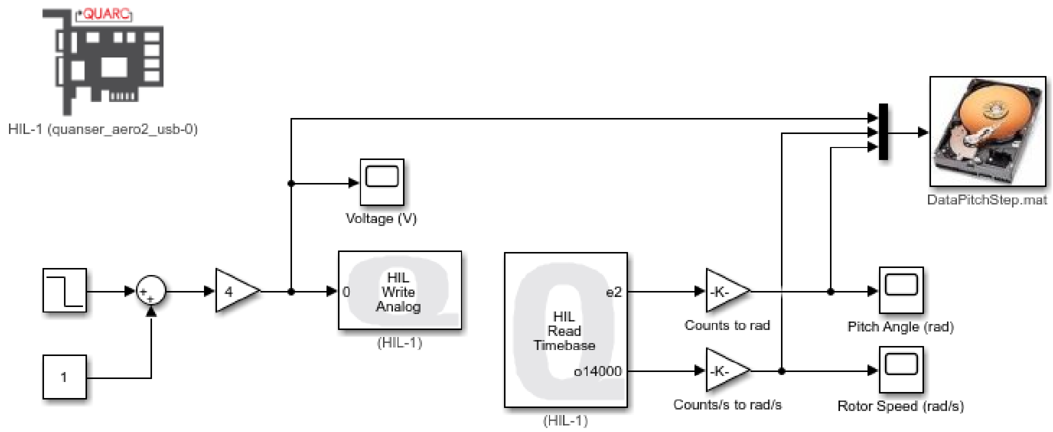


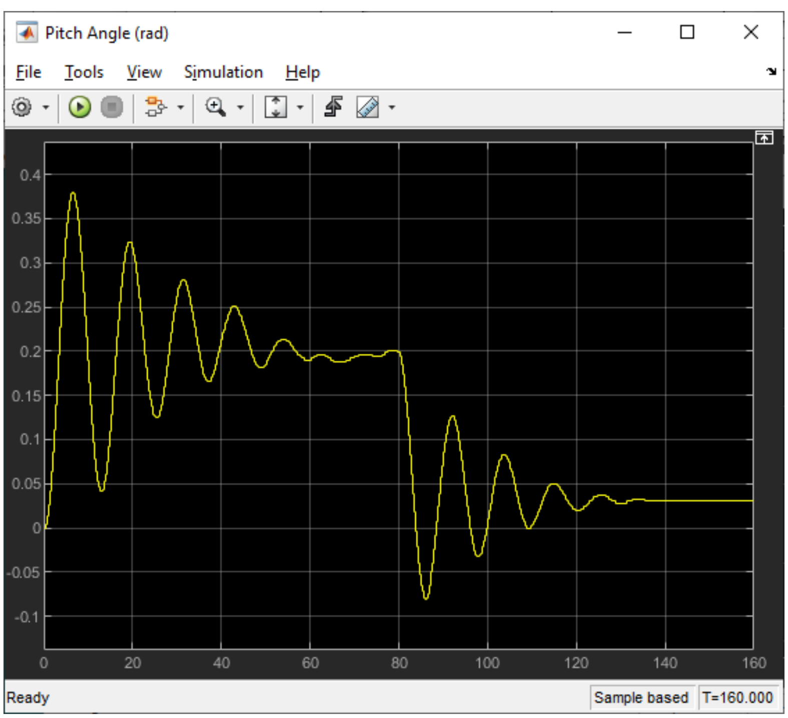

Figure 1 - Simulink model used with QUARC to apply a voltage to the front pitch rotor and measure the pitch angle.

The Simulink model uses the `HIL Write Analog` and `HIL Read Timebase` blocks from the *QUARC Targets* library to apply a voltage to the front rotor and measure the corresponding pitch angle response. The response is saved into a MATLAB *.mat files using the `To Host File` block. This can then be used to plot the results and perform the parameter estimation analysis.

Build and run the following Simulink model in QUARC by clicking on the *Monitor & Tune* button.

```
% Load Simulink model
open("q_aero2_pitch_step.slx");
```

The scopes shown in Figure 2 show the pitch angle response from applying a voltage to the front pitch rotor.



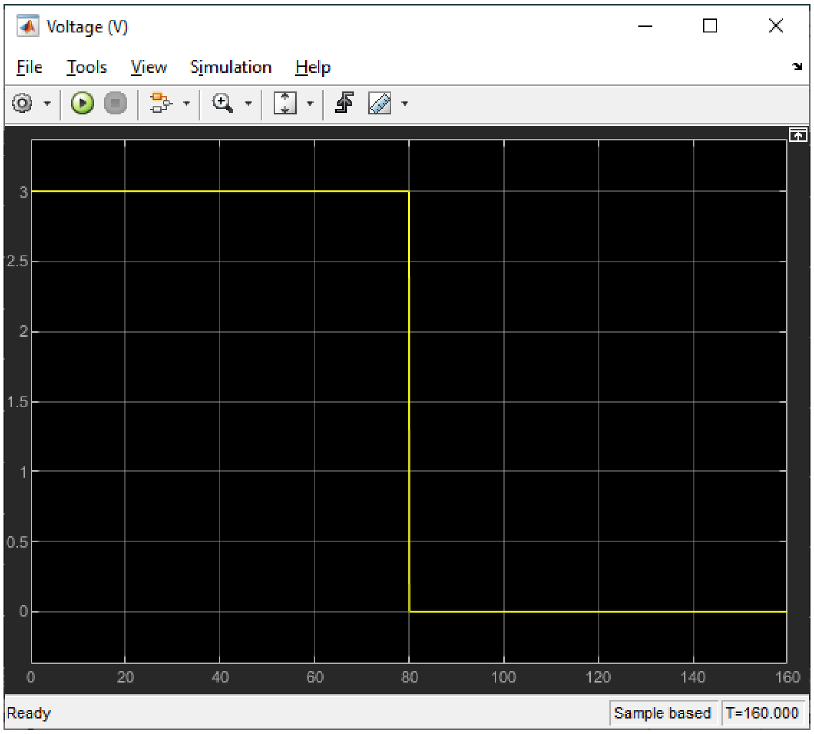
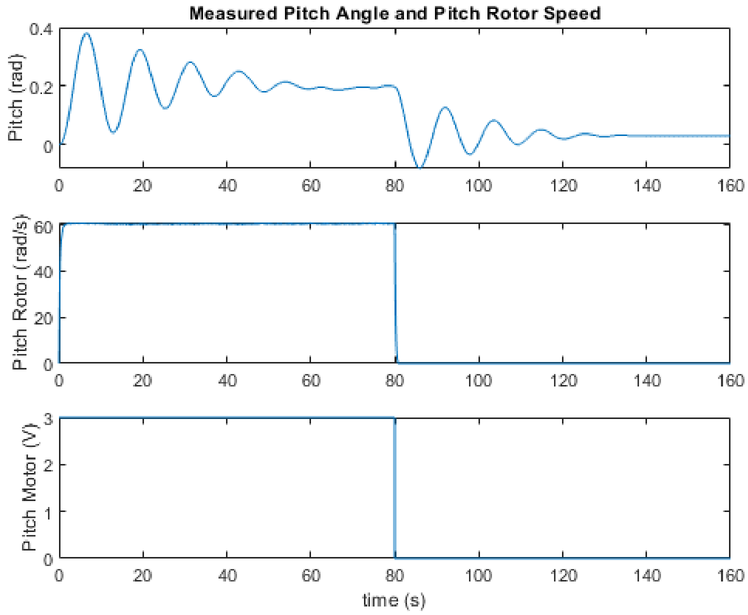



Figure 2 - Pitch angle response from applying a step to the front pitch rotor.

Plot measured response

```
% Load measured data from past run
load('DataPitchStep.mat');
% store in variables
t = DataPitchStep(1,:); % time (s)
Vp = DataPitchStep(2,:); % pitch voltage (V)
wm_p = DataPitchStep(3,:); % rotor speed (rad/s)
theta = DataPitchStep(4,:); % pitch angle (rad)
%
subplot(3,1,1);
plot(t,theta);
title('Measured Pitch Angle and Pitch Rotor Speed');
ylabel('Pitch (rad)');
subplot(3,1,2);
plot(t,wm_p);
ylabel('Pitch Rotor (rad/s)');
subplot(3,1,3);
plot(t,Vp);
ylabel('Pitch Motor (V)');
xlabel('time (s)');
```




## Lab 2: Parameter Estimation

### Finding the Pitch Stiffness and Damping

The prototype second-order transfer function when there is a steady-state value of $K_{ss}$ can be represented by the following

$$\frac{Y(s)}{U(s)} = \frac{K_{ss}\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2}$$

where $\omega_n$ is the natural frequency and $\zeta$ is the damping ratio. Based on the plant transfer function, we can find the stiffness and viscous damping parameters using the equations

$$K_{sp} = J_p\omega_n^2$$

$$D_p = 2J_p\zeta\omega_n$$

The parameters can be calculated by measuring the natural frequency and damping ratio of the free-oscillation response.

### Finding the Natural Frequency

The period of the oscillations in a system response can be found using the equation

$$T_{osc} = \frac{t_n - t_1}{n - 1}$$

where $t_n$ is the time of the $n^{th}$ oscillation, $t_1$ is the time of the first peak, and $n$is the number of oscillations considered. From this, the damped natural frequency (in radians per second) is

$$\omega_d = \frac{2\pi}{T_{osc}}$$

and the undamped natural frequency is

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}}.$$

**Finding the Damping Ratio**

The damping ratio of a second-order system can be found from its response. For a typical second-order underdamped system, the subsidence ratio (i.e., decrement ratio) is defined as

$$\delta = \frac{1}{n}\ln\frac{O_1}{O_n}$$

where $O_1$ is the peak of the first oscillation and $O_n$ is the peak of the $n^{th}$ oscillation. Note that $O_1 > O_n$, as this is a decaying response.

The damping ratio is defined

$$\zeta = \frac{1}{\sqrt{1+\left(\frac{2\pi}{\delta}\right)^2}}$$

For more information, please see the Wikipedia reference.

Load the AERO 2 Parameters

```
aero2_parameters;
```

Enter the peak value and time of the first (largest) overshoot and the last overshoot (n^th) of the *free-oscillation response*.

```
% Measure first overshoot
O1 = 0.127; % 0.218;
t1 = 92.2; % 71.25;
% Measure nth overshoot
n = 4; % 8;
On = 3.71-2; % 0.0131;
tn = 125.7; % 142.6;
```

Calculate the natural frequency and damping ratio of the free-oscillation response.

```
% Oscillation period (s)
Tosc = (tn-t1) / (n-1);
% Logarithmic decrement (i.e., subsidence ratio)
sigma = 1/n*log(O1/On);
%
% Damping ratio
zeta = 1 / sqrt(1 + (2*pi/sigma)^2);
```

```
% Damped natural frequency (rad/s)
wd = 2*pi/Tosc;
% Undamped natural frequency (rad/s)
wn = wd / sqrt(1-zeta^2);
```

Calculate siffness and damping based on measured response.

```
% Stiffness (N.m/rad)
Ksp = Jp*wn^2
```

```
Ksp = 0.0074
```

```
% Damping (N.m/(rad/s))
Dp = 2*zeta*wn*Jp
```

```
Dp = 0.0027
```

Based on the given moment of inertia, the stiffness and damping of the 1 DOF pitch system are: $K_{sp} = 0.00884$ N-m/rad and $D_p = 0.00160$ N-m/(rad/s).

## Finding the Thrust

Applying a step input voltage to the pitch rotor speed with an amplitude of $V_{p0}$, the open-loop transfer function model becomes:

$$\Theta(s) = \frac{\frac{D_t K_{pp}}{J_p}}{s^2 + \frac{D_p}{J_p}s + \frac{K_{sp}}{J_p}} \frac{V_{p0}}{s}$$

Applying the Final-Value Theorem (FVT) on the pitch transfer function, the steady-state angle

$$\theta_{ss} = \lim_{s \to 0} \frac{\frac{D_t K_{pp}}{J_p}}{s^2 + \frac{D_p}{J_p}s + \frac{K_{sp}}{J_p}} V_{p0} = \frac{D_t K_{pp}}{K_{sp}} V_{p0}$$

Solving for the thrust

$$K_{pp} = \frac{\theta_{ss} K_{sp}}{D_t V_{p0}}$$

Find pitch thrust from step response

```
% Steady-state pitch angle at end of step input 120 sec (rad)
theta_ss_pp = 0.200; % 0.293;
% Rotor voltage at same time (V)
Vm_ss_pp = max(Vp);
% Pitch thrust gain (V)
Kpp = theta_ss_pp * Ksp / (Dt * Vm_ss_pp)
```

```
Kpp = 0.0030
```

Based on this experiment, the rotor thrust constant is $K_{pp} = 0.00387$ N/V.

## Lab 3: Model Validation

The model can be validated by running the hardware in parallel with the identified model using the `q_aero2_pitch_model_val.slx` Simulink model in QUARC.

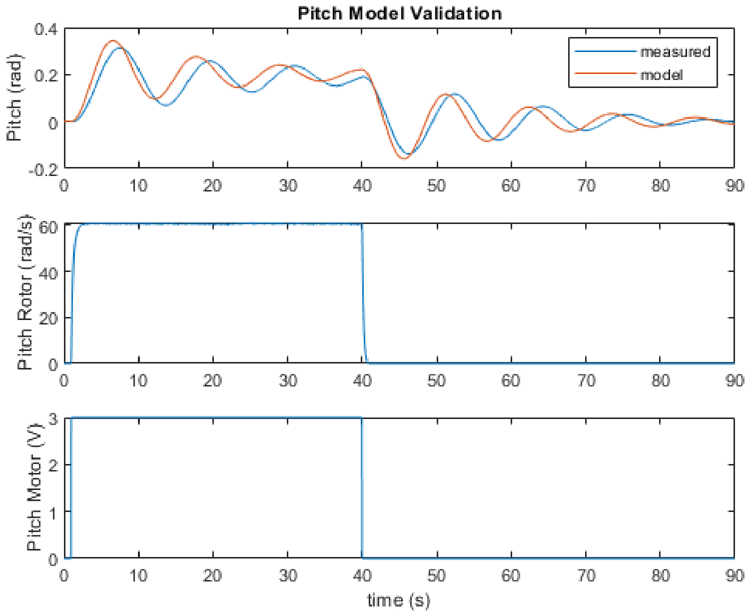
To do this:

1. Make sure with the identified parameters `Kpp`, `Dp`, and `Ksp` are loaded.
2. Build and run the following Simulink model in QUARC by clicking on the *Monitor & Tune* button.

```
% validate new model
open("q_aero2_pitch_model_val.slx")
```

Plot results.

```
% load saved data from MAT files
load("DataPitchModelVal.mat");
%
t = DataPitchModelVal(1,:); % time (s)
Vp = DataPitchModelVal(2,:); % pitch rotor voltage (V)
wm = DataPitchModelVal(3,:); % pitch rotor speed (rad/s)
theta_meas = DataPitchModelVal(4,:); % measured pitch angle from hardware (rad)
theta_sim = DataPitchModelVal(5,:);  % simulated pitch angle from model (rad)
%
subplot(3,1,1);
plot(t,theta_meas,t,theta_sim);
title('Pitch Model Validation');
ylabel('Pitch (rad)');
legend('measured','model')
subplot(3,1,2);
plot(t,wm);
ylabel('Pitch Rotor (rad/s)');
subplot(3,1,3);
plot(t,Vp);
ylabel('Pitch Motor (V)');
xlabel('time (s)');
```




The model does not quite match the response measured from the Aero 2 hardware. As mentioned in *Aero2_2DOF_Modeling*, the model used to represent the motion of the Aero 2 pitch is an approximation. The actual system is higher-order and includes other aerodynamics that are not included. More accurate values could be obtained by using system identification software or other techniques, e.g., least-squares method.