Pitch Thrust from Yaw - Parameter Estimation

There is a reaction torque acting on the pitch from the yaw rotor. This lab shows how to find the thrust gain parameter that captures this effect.

Concept Review

In the 2 DOF configuration, the motion of the pitch is affected by both rotors. The pitch or front rotor applies a direct thrust about the pitch axis but the yaw rotor generates a torque on the pitch axis as well. This can be represented by the following equation of motion (EOM):

$$J_p\ddot{\theta} + D_p\dot{\theta} + K_{sp}\theta = K_{pp}D_tV_p + K_{py}D_tV_y$$

where θ is the pitch angle, J_p is the equivalent moment of inertia acting about the pitch axis, D_p is the viscous damping, K_{sp} is the stiffness, K_{pp} is the force thrust gain acting on the pitch from the pitch rotor, K_{py} is the thrust froce gain acting on the pitch from the yaw rotor, D_t is the distance from the pivot point to the propeller, and V_p is the voltage applied to the front pitch rotor motor.

The pitch thrust K_{pp}, inertia, damping, and stiffness were identified in the *Pitch Parameter Estimation* lab. Because the yaw-axis is locked and we are only applying a thrust to the rear/yaw rotor, $V_p = 0$, and the EOM becomes

$$J_p\ddot{\theta} + D_p\dot{\theta} + K_{sp}\theta = K_{py}D_tV_y$$

Given that the system starts at rest, the initial condition are zero and we have the following transfer function

$$\Theta(s) = \frac{K_{py}D_tV_y}{J_ps^2 + D_ps + K_{sp}}$$

We can use this transfer function to identify that pitch from yaw force gain K_{py}.

Lab Procedure

Apply a voltage to the rear/yaw rotor and examine the reponse in the pitch to measure the thrust from the reaction torque.

Setup

1. Make sure the Aero 2 has been tested as instructed in the Quick Start Guide.
2. Launch MATLAB and browse to the working directory that includes the Simulink models for this lab.
3. Configure the Aero 2 in the 2 DOF Helicopter configuration:
4. **Unlock** the pitch axis and **lock** the yaw axis.
5. Rear rotor 1 is **vertical** and front rotor 0 is **horizontal**.
6. Mount weight on each rotor.
7. Connect the USB cable to your PC/laptop.
8. Connect the power and turn the power switch ON. The Aero base LED should be red.

Lab 1: Get Open-Loop Response

The open-loop response can be obtained by running the `q_aero2_pitch_from_yaw_step` Simulink model shown in Figure 1 with QUARC.

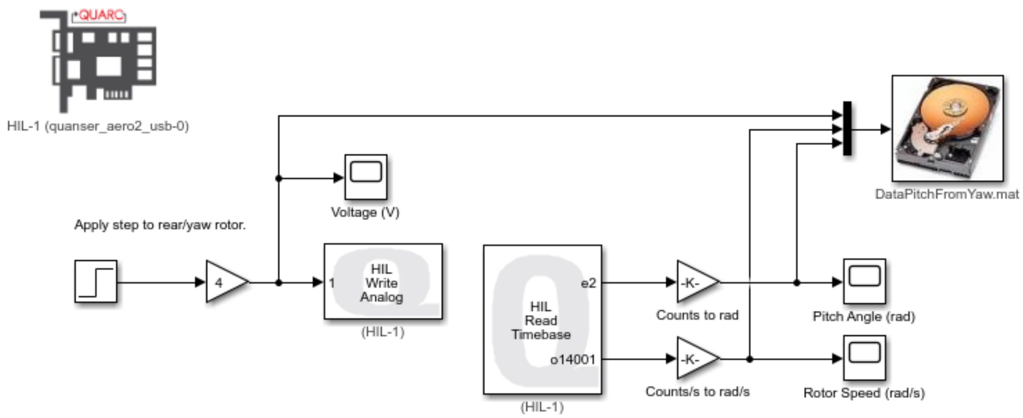


Figure 1 - Simulink model used with QUARC to apply a step to the rear/yaw rotor and measure the pitch angle.

The Simulink model uses the `HIL Write Analog` and `HIL Read Timebase` blocks from the *QUARC Targets* library to apply a voltage to the rear yaw rotor and measure the corresponding pitch angle response. The response is saved into a MATLAB *.mat files using the `To Host File` block. This can then be used to plot the results and perform the parameter estimation analysis.

Build and run the following Simulink model in QUARC by clicking on the *Monitor & Tune* button.

```
% Load Simulink model
open("q_aero2_pitch_from_yaw_step.slx");
```

Figure 2 shows the pitch angle response and applied voltage in the scopes.

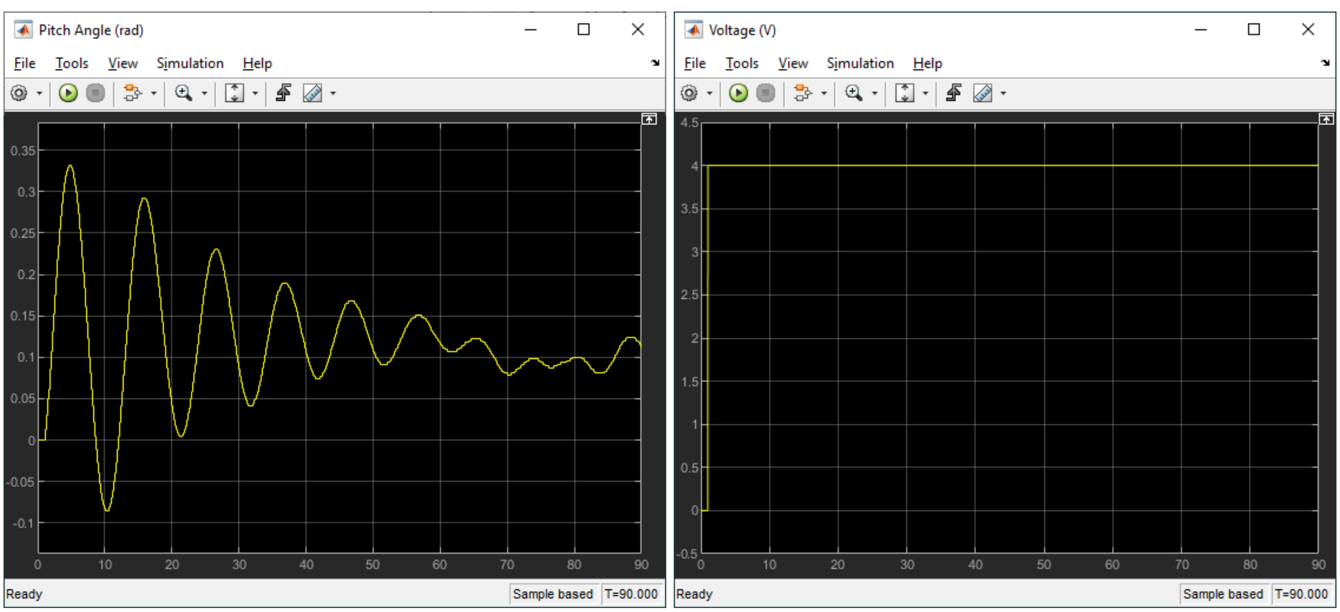
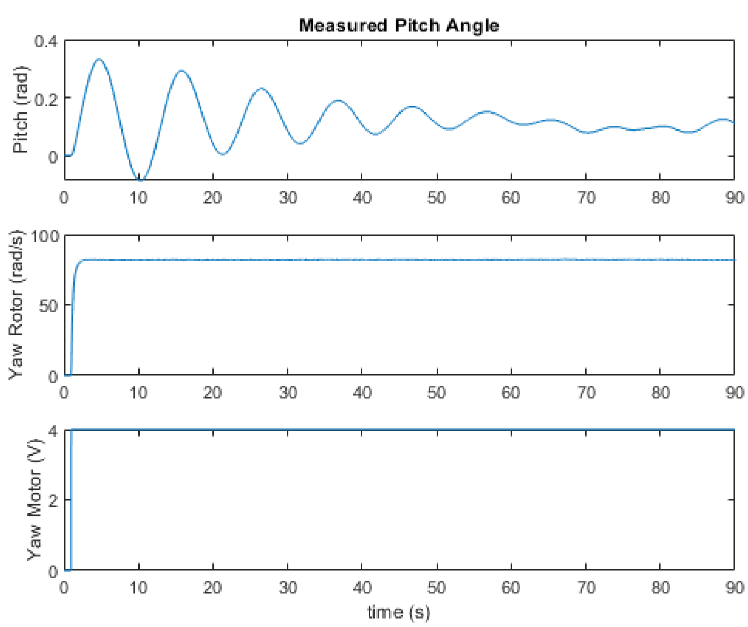


Figure 2 - Pitch angle response from applying a step to the rear yaw rotor.

Plot measured response

```
% Load measured data from past run
load('DataPitchFromYaw.mat');
% store in variables
t = PitchFromYaw(1,:); % time (s)
Vy = PitchFromYaw(2,:); % yaw/rear motor voltage (V)
wm_y= PitchFromYaw(3,:); % yaw/rear rotor speed (rad/s)
theta = PitchFromYaw(4,:); % pitch angle (rad)
%
subplot(3,1,1);
plot(t,theta);
title('Measured Pitch Angle');
ylabel('Pitch (rad)');
subplot(3,1,2);
plot(t,wm_y);
ylabel('Yaw Rotor (rad/s)');
subplot(3,1,3);
plot(t,Vy);
ylabel('Yaw Motor (V)');
xlabel('time (s)');
```



Lab 2: Finding the Thrust Gain

Applying a step input with an amplitude of V_{y0} to the yaw rotor

$$\Theta(s) = \frac{\dfrac{D_t K_{py}}{J_p}}{s^2 + \dfrac{D_p}{J_p}s + \dfrac{K_{sp}}{J_p}} \frac{V_{y0}}{s}$$

Using the Final-Value Theorem (FVT), the steady-state angle is

$$\theta_{ss} = \lim_{s \to 0} \frac{\dfrac{D_t K_{py}}{J_p}}{s^2 + \dfrac{D_p}{J_p}s + \dfrac{K_{sp}}{J_p}} V_{y0} = \frac{D_t K_{py}}{K_{sp}} V_{y0}$$

Solving for the thrust

$$K_{py} = \frac{\theta_{ss} K_{sp}}{D_t V_{y0}}$$

Load the AERO 2 Parameters

```
aero2_parameters;
```

Load stiffness found in the *Pitch Parameter Estimation* lab.

```
% Stiffness (N-m/V)
Ksp = 0.00884;
```

Find the pitch from yaw thrust force gain.

```
% steady-state pitch angle (rad)
theta_ss = 0.122;
% yaw step input (V)
Vp0 = max(Vy);
% pitch thrust from yaw (N/V)
Kpy = theta_ss * Ksp / Dt / Vp0
```

```
Kpy = 0.0016
```

The measured cross-thrust acting on the pitch-axis from the yaw/back rotor is

$K_{py} = 4.0259 \times 10^{-4}$ N/V