# Yaw Parameter Estimation

This lab shows how to find the thrust gain and damping parameter of the yaw-only system (i.e., pitch is locked).

## Concept Review

The AERO 2 yaw-axis only model can be represented by the equation of motion

$$J_y\ddot{\psi}(t) + D_y\dot{\psi}(t) = K_{yy}D_tV_y$$

where $\psi$ is the yaw angle, $J_y$ is the equivalent moment of inertia acting about the yaw axis, $D_y$ is the viscous damping, $K_{yy}$ is the force thrust gain, $D_t$ is the distance from the pivot point to the propeller, $V_y$ is the voltage applied to the rear/yaw rotor motor.

Based on the EOM, we have the folloiwng transfer function

$$\Psi(s) = \frac{K_{yy}D_tV_y(s)}{J_ys^2 + D_ys}$$

In this lab we apply a yaw step voltage to measure yaw thrust, $K_{yy}$, and damping, $D_y$. The moment of inertia, $J_y$, is known.

## Lab Procedure

Apply a step voltage to the rear.yaw rotor to measure the thrust gain and then set it to 0V to measure the free-oscillation decaying response to measure the damping.

### Setup

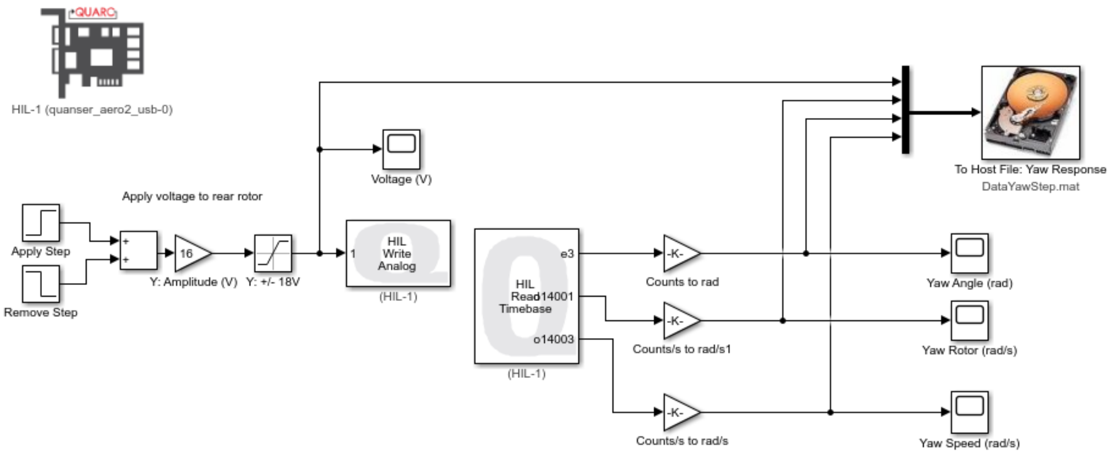
1. Make sure the Aero 2 has been tested as instructed in the Quick Start Guide.
2. Launch MATLAB and browse to the working directory that includes the Simulink models for this lab.
3. Configure the Aero 2 in the 2 DOF Helicopter configuration:
4. **Lock** the pitch axis and **unlock** the yaw axis.
5. Rear rotor 1 is **vertical** and front rotor 0 is **horizontal**.
6. Mount weight on each rotor.
7. Connect the USB cable to your PC/laptop.
8. Connect the power and turn the power switch ON. The Aero base LED should be red

Load the AERO 2 Parameters

```
aero2_parameters;
```

### Lab 1: Get Open-Loop Response

The open-loop response can be obtained by running the `q_aero2_yaw_step` Simulink model shown below in QUARC.




The Simulink model uses the `HIL Write Analog` and `HIL Read Timebase` blocks from the *QUARC Targets* library to apply a voltage to rear yaw rotor and measure the corresponding yaw angle and rate response. The response is saved into a MATLAB *.mat files using the `To Host File` block. This can then be used to plot the results and perform the parameter estimation analysis.

Build and run the following Simulink model in QUARC by clicking on the *Monitor & Tune* button.

```
% Load Simulink model
open("q_aero2_yaw_step.slx");
```

Figure 2 shows the yaw angle response and applied voltage in the scopes.

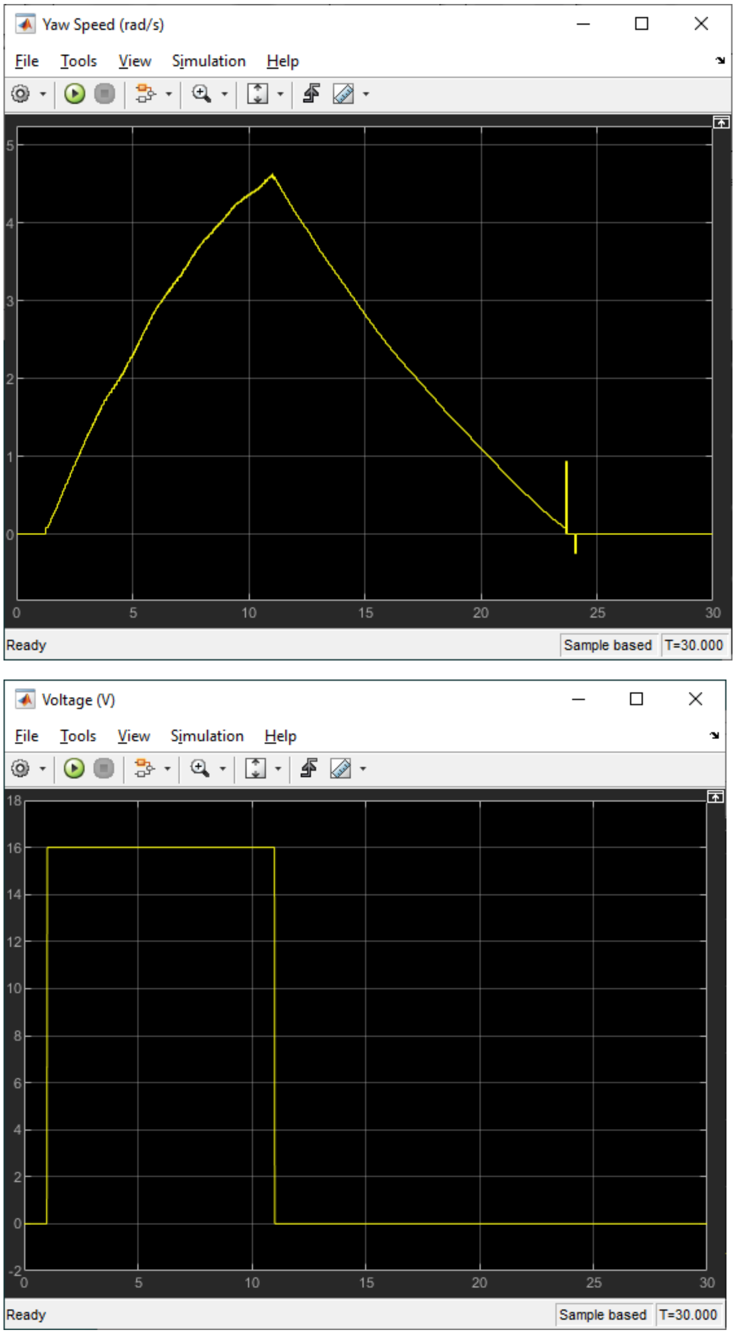
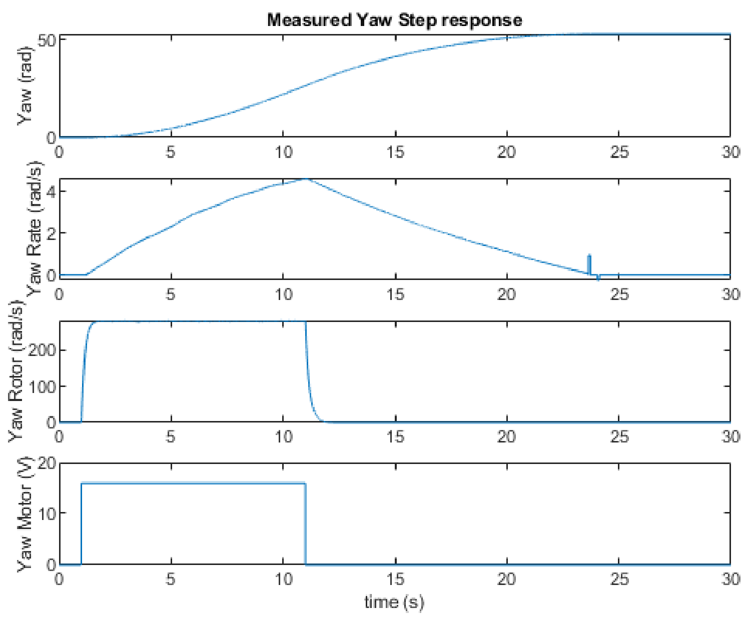



Figure 2 - Yaw angle rate response from applying a step to the rear yaw rotor.

Plot measured response

```
% Load measured data from past run
load("DataYawStep.mat");
% store in variables
t = DataYawStep(1,:); % time (s)
Vy = DataYawStep(2,:); % yaw/rear motor voltage (V)
wm_y = DataYawStep(3,:); % rotor speed (rad/s)
psi = DataYawStep(4,:); % yaw angle (rad)
psi_dot = DataYawStep(5,:);  % yaw rate (rad)
figure;
```

```
subplot(4,1,1);
plot(t,psi);
title('Measured Yaw Step response');
ylabel('Yaw (rad)');
subplot(4,1,2);
plot(t,psi_dot);
ylabel('Yaw Rate (rad/s)');
subplot(4,1,3);
plot(t,wm_y);
ylabel('Yaw Rotor (rad/s)');
subplot(4,1,4);
plot(t,Vy);
ylabel('Yaw Motor (V)');
xlabel('time (s)');
```




## Lab 2: Parameter Estimation

The thrust force gain can be found from the step response and the damping can be found from the free-oscillation response (i.e., after the step is applied).

### Finding the Damping

In terms of angular rate, the equation of motion of the yaw becomes

$$J_y\dot{\beta}(t) + D_y\beta(t) = 0$$

where $\beta(t) = \dot{\psi}(t)$. Taking its Laplace transform

$$J_p(\beta(s)s - \beta(0^-)) + D_y\beta(s) = 0$$

and solving for the speed we get

$$\beta(s) = \frac{J_y}{J_y s + D_y}\beta(0^-) = \frac{J_y/D_y}{J_y/D_y s + 1}\beta(0^-).$$

The yaw free-oscillation transfer function matches the prototype first-order transfer function. Based on the measured time constant of the response, its damping can be found with

$$D_y = \frac{J_y}{\tau}$$

### First-Order Response

To find the time constant from first-order response from an step as depicted in in Figure 1, find the time it takes for the response to reach $e^{-1}$ or 37% of its final steady-state value.

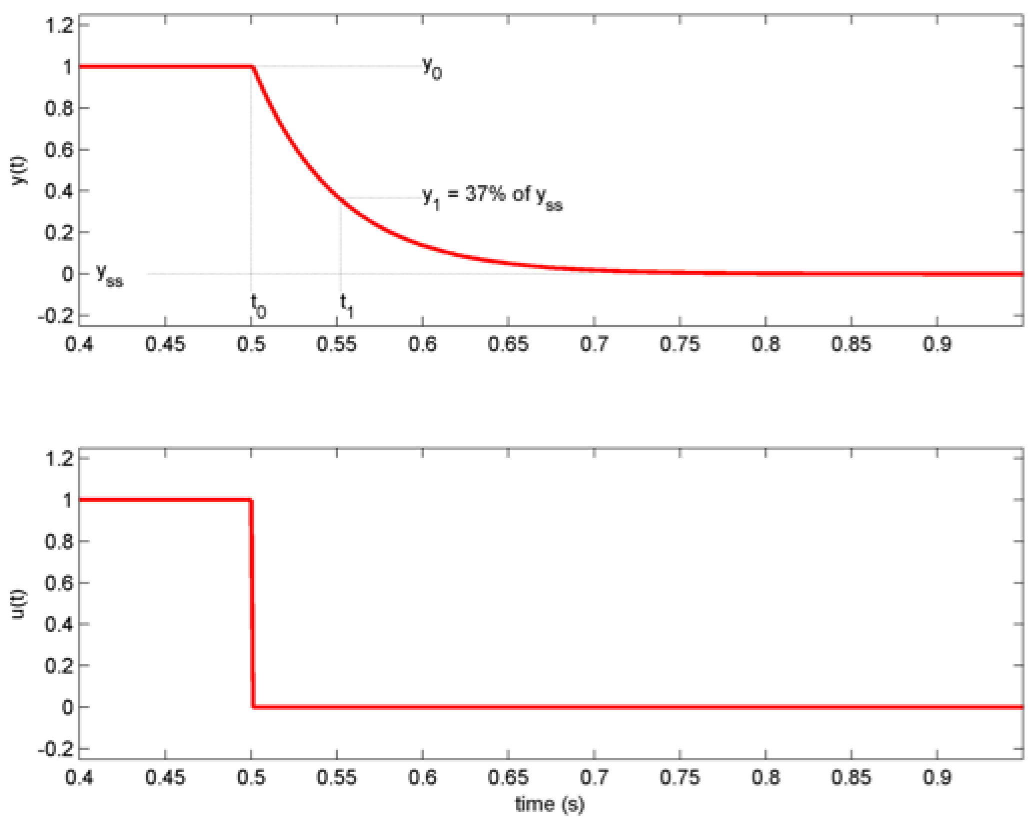



Figure 1: First-order step decaying response

In this case we need to find

$$y(t_1) = y_1 = e^{-1}(y_0 - y_{ss})$$

and the time constant is $\tau = t_1 - t_0$ .

```
% Voltage applied (V)
Vy0 = max(Vy);
% time when step stops (s)
tstop = 11;
```

```
% sampling interval (s)
h = 0.002;
% index when step stops
istop = ceil(tstop/h)+1;
% yaw rate at end of step (rad/s)
w0 = psi_dot(istop);
w_3db = exp(-1)*w0; % 36% of initial value
i_3db = find(psi_dot > w_3db, 1, 'last' );
t_3db = i_3db * h;
tau_y = t_3db-tstop; % time constant (s)
Dy = Jy/tau_y
```

```
Dy = 0.0033
```

### Find the Thrust Gain

The equation of motion when applying a step voltage to the rear/yaw rotor is

$$J_y\dot{\beta} + D_y\beta = K_{yy}D_tV_y$$

Solving for the thrust gain parameter

$$K_{yy} = \frac{J_y\dot{\beta} + D_y\beta}{D_tV_y}$$

We can find the thrust gain from the measure yaw rate and derived acceleration

```
% Sampling interval of controller (s)
h = 0.002;
% Yaw acceleration from yaw rate measurement (rad/s^2)
psi_ddot = diff(psi_dot)/h;
% Thrust gain (N/V)
Kyy = ( Jy*psi_ddot(1:istop) + Dy*psi_dot(1:istop) ) / ( Dt*Vy(1:istop) )
```

```
Kyy = 0.0074
```

The identified damping and yaw thrust gain are: $D_y = 0.00334$ N-m/(rad/s) and $K_{yy} = 0.0074$N/V.