

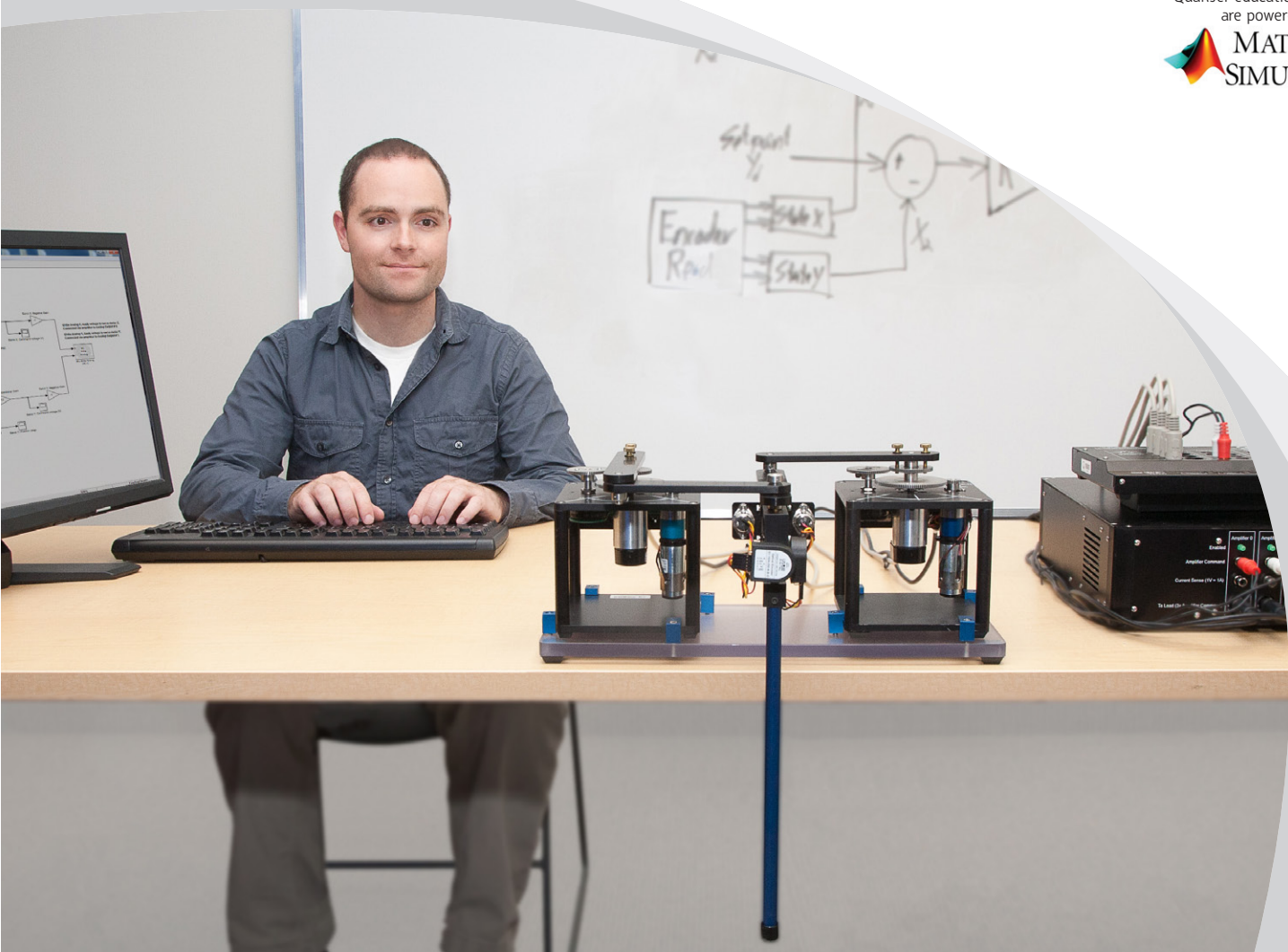


# LABORATORY GUIDE

## 2 DOF Gantry Experiment for MATLAB®/Simulink® Users

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# 1 INTRODUCTION

The challenge of this experiment is to design a feedback controller that dampens out the swinging of a single pendulum load suspended from a two degree of freedom (2 DOF) joint.

The 2 DOF Gantry setup is based on the 2 DOF Robot configuration. For more information on the 2 DOF Robot, see the 2 DOF Robot User Manual [7]. The 2 DOF joint module attaches underneath the end-effector of the four-bar linkage system. The module consists of an instrumented 2 DOF joint to which a 12-inch rod is mounted and free to swing about two orthogonal axes. The controller must reduce the swinging of the load when it is disturbed or when the 2 DOF robot is commanded to move the load in its workspace.

A few real-world applications of the gantry problem include, for example, a crane lifting and moving a heavy payload, or a pick-and-place gantry robot of an assembly line

The laboratory objective is to design two independent state-feedback controllers, using LQR, for the two actuated rotary joints which should reduce the swinging of the load (pendulum) attached to the 2 DOF joint.

## Topics Covered

- Obtain a state-space representation of the open-loop system.
- Design and tune an LQR-based state-feedback controller satisfying the closed-loop system's desired design specifications.
- Simulate the system and ensure it meets the given specifications.
- Implement your state-feedback controller on the 2 DOF Gantry system and evaluate its actual performance.
- Examine the disturbance response of the stabilized suspended-pendulum-robot system, in response to a tap to the pendulum.

## Prerequisites

In order to successfully carry out this laboratory, the user should be familiar with the following:

1. See the system requirements in Section 4 for the required hardware and software.
2. Modeling and state-space representation.
3. State-feedback design using Linear-Quadratic Regular (LQR) optimization.
4. Basics of [Simulink®](#).
5. QUARC Integration lab detailed in Appendix A in the SRV02 Workbook [5].

## 2 BACKGROUND

The 2 DOF Gantry is modeled as two de-coupled rotary gantry systems. The linearized model is then used with the Linear-Quadratic Regular (LQR) technique to design the feedback gain.

### 2.1 Modeling

If the end-effector does not deviate greatly from the zero or home position, we assume that the coupling between the two axes is low. With this assumption, the system is modeled as two 1 DOF rotary gantry systems where the x-axis servo controls the gantry angle in the  $x$  direction and the y-axis servo controls the gantry angle in the  $y$  direction. This section will summarize how to model a single rotary gantry system.

#### 2.1.1 Model Convention

The rotary pendulum model is shown in Figure 2.1. The rotary arm pivot is attached to the SRV02 system and is actuated. The arm has a length of  $L_r$ , a moment of inertia of  $J_r$ , and its angle,  $\theta$ , increases positively when it rotates counter-clockwise (CCW). The servo (and thus the arm) should turn in the CCW direction when the control voltage is positive, i.e.,  $V_m > 0$ .

The pendulum link is connected to the end of the rotary arm. It has a total length of  $L_p$  and its center of mass is  $\frac{L_p}{2}$ . The moment of inertia about its center of mass is  $J_p$ . The inverted pendulum angle,  $\alpha$ , is zero when it is perfectly upright in the vertical position and increases positively when rotated CCW.

#### 2 DOF Gantry HOME Position

Recall from the 2 DOF Robot User Manual [7] that the home position of the 2 DOF Robot is represented by the schematic illustrated in Figure 2.2. This is defined when both actuated servo joints are zero, i.e.,  $\theta_x = 0$  and  $\theta_y = 0$ . In the home position, the 2-DOF Gantry is assumed to be suspended underneath the end-effector at point E and motionless. Thus the angle of both orthogonal gantry joints are defined  $\alpha_x = 0$  and  $\alpha_y = 0$ .

#### 2.1.2 Nonlinear Equations of Motion

Instead of using classical mechanics, the Lagrange method is used to find the equations of motion of the system. This systematic method is often used for more complicated systems such as robot manipulators with multiple joints.

More specifically, the equations that describe the motions of the rotary arm and the pendulum with respect to the servo motor voltage, i.e. the dynamics, will be obtained using the Euler-Lagrange equation:

$$\frac{\partial^2 L}{\partial t \partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i$$

The variables  $q_i$  are called *generalized coordinates*. For this system let

$$q(t)^\top = [\theta(t) \ \alpha(t)] \quad (2.1)$$

where, as shown in Figure 2.1,  $\theta(t)$  is the rotary arm angle and  $\alpha(t)$  is the inverted pendulum angle. The corresponding velocities are

$$\dot{q}(t)^\top = \left[ \frac{\partial \theta(t)}{\partial t} \quad \frac{\partial \alpha(t)}{\partial t} \right]$$

**Note:** The dot convention for the time derivative will be used throughout this document, i.e.,  $\dot{\theta} = \frac{d\theta}{dt}$ . The time variable  $t$  will also be dropped from  $\theta$  and  $\alpha$ , i.e.,  $\theta = \theta(t)$  and  $\alpha = \alpha(t)$ .

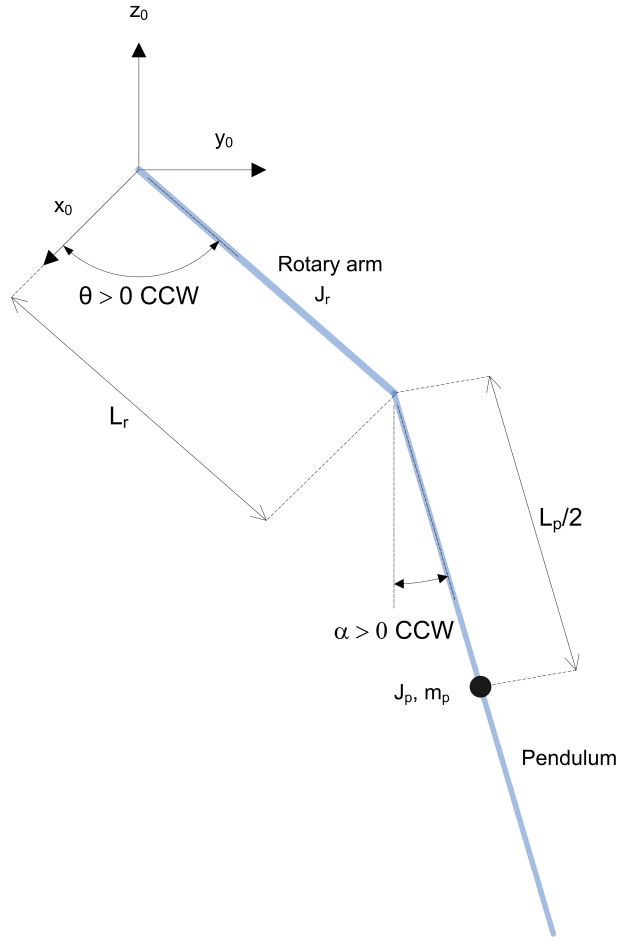


Figure 2.1: Rotary gantry conventions

With the generalized coordinates defined, the Euler-Lagrange equations for the rotary pendulum system are

$$\begin{aligned}\frac{\partial^2 L}{\partial t \partial \dot{\theta}} - \frac{\partial L}{\partial \theta} &= Q_1 \\ \frac{\partial^2 L}{\partial t \partial \dot{\alpha}} - \frac{\partial L}{\partial \alpha} &= Q_2\end{aligned}$$

The Lagrangian of the system is described

$$L = T - V$$

where  $T$  is the total kinetic energy of the system and  $V$  is the total potential energy of the system. Thus the Lagrangian is the difference between a system's kinetic and potential energies.

The generalized forces  $Q_i$  are used to describe the non-conservative forces (e.g., friction) applied to a system with respect to the generalized coordinates. In this case, the generalized force acting on the rotary arm is

$$Q_1 = \tau - D_r \dot{\theta}$$

and acting on the pendulum is

$$Q_2 = -D_p \dot{\alpha}.$$

See [3] for a description of the corresponding SRV02 parameters (e.g. such as the back-emf constant,  $k_m$ ). Our control variable is the input servo motor voltage,  $V_m$ . Opposing the applied torque is the viscous friction torque, or viscous damping, corresponding to the term  $D_r$ . Since the pendulum is not actuated, the only force acting on the link is the damping. The viscous damping coefficient of the pendulum is denoted by  $D_p$ .

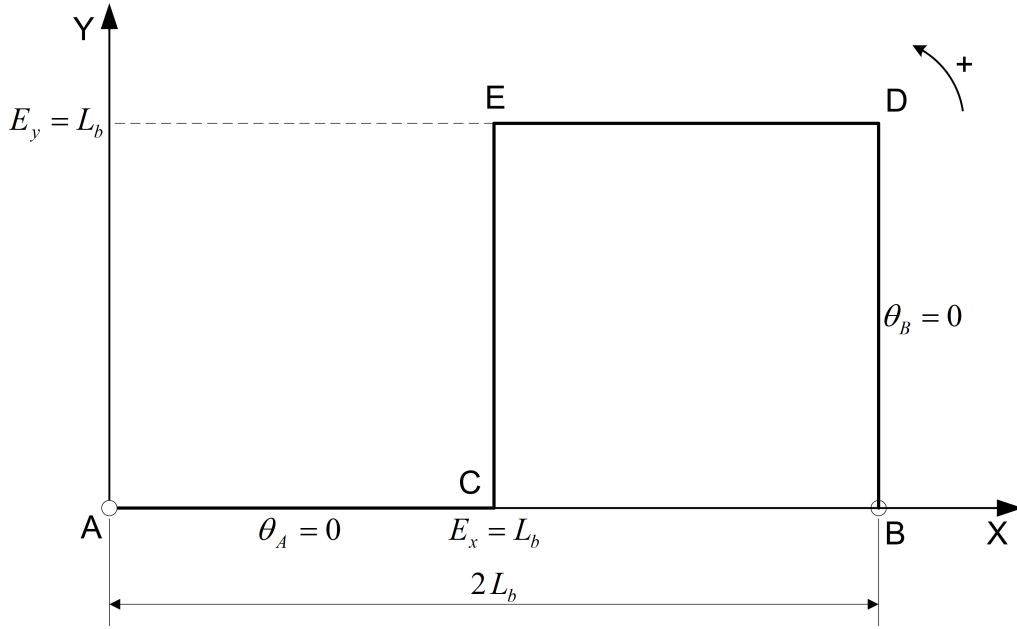


Figure 2.2: 2 DOF Robot HOME Position.

The Euler-Lagrange equations is a systematic method of finding the equations of motion, i.e., EOMs, of a system. Once the kinetic and potential energy are obtained and the Lagrangian is found, then the task is to compute various derivatives to get the EOMs. After going through this process, the nonlinear equations of motion for the SRV02 rotary pendulum are:

$$\left( m_p L_r^2 + \frac{1}{4} m_p L_p^2 - \frac{1}{4} m_p L_p^2 \cos(\alpha)^2 + J_r \right) \ddot{\theta} - \left( \frac{1}{2} m_p L_p L_r \cos(\alpha) \right) \ddot{\alpha} + \left( \frac{1}{2} m_p L_p^2 \sin(\alpha) \cos(\alpha) \right) \dot{\theta} \dot{\alpha} + \left( \frac{1}{2} m_p L_p L_r \sin(\alpha) \right) \dot{\alpha}^2 = \tau - D_r \dot{\theta} \quad (2.2)$$

$$\frac{1}{2} m_p L_p L_r \cos(\alpha) \ddot{\theta} + \left( J_p + \frac{1}{4} m_p L_p^2 \right) \ddot{\alpha} - \frac{1}{4} m_p L_p^2 \cos(\alpha) \sin(\alpha) \dot{\theta}^2 + \frac{1}{2} m_p L_p g \sin(\alpha) = -D_p \dot{\alpha}. \quad (2.3)$$

The torque applied at the base of the rotary arm (i.e., at the load gear) is generated by the servo motor as described by the equation

$$\tau = \frac{\eta_g K_g \eta_m k_t (V_m - K_g k_m \dot{\theta})}{R_m}. \quad (2.4)$$

See [3] for a description of the corresponding SRV02 parameters (e.g. such as the back-emf constant,  $k_m$ ).

Both the equations match the typical form of an EOM for a single body:

$$J\ddot{x} + b\dot{x} + g(x) = \tau_1$$

where  $x$  is an angular position,  $J$  is the moment of inertia,  $b$  is the damping,  $g(x)$  is the gravitational function, and  $\tau_1$  is the applied torque (scalar value).

For a generalized coordinate vector  $q$ , this can be generalized into the matrix form

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau \quad (2.5)$$

where  $D$  is the inertial matrix,  $C$  is the damping matrix,  $g(q)$  is the gravitational vector, and  $\tau$  is the applied torque vector.

The nonlinear equations of motion given in 2.2 and 2.3 can be placed into this matrix format.

### 2.1.3 Linearizing

Here is an example of how to linearize a two-variable nonlinear function called  $f(z)$ . Variable  $z$  is defined

$$z^\top = [z_1 \ z_2]$$

and  $f(z)$  is to be linearized about the operating point

$$z_0^\top = [a \ b]$$

The linearized function is

$$f_{lin} = f(z_0) + \left( \frac{\partial f(z)}{\partial z_1} \right) \bigg|_{z=z_0} (z_1 - a) + \left( \frac{\partial f(z)}{\partial z_2} \right) \bigg|_{z=z_0} (z_2 - b)$$

### 2.1.4 Linear State-Space Model

The linear state-space equations are

$$\dot{x} = Ax + Bu \quad (2.6)$$

and

$$y = Cx + Du \quad (2.7)$$

where  $x$  is the state,  $u$  is the control input,  $A$ ,  $B$ ,  $C$ , and  $D$  are state-space matrices. For the rotary pendulum system, the state and output are defined

$$x^\top = [\theta \ \alpha \ \dot{\theta} \ \dot{\alpha}] \quad (2.8)$$

and

$$y^\top = [x_1 \ x_2]. \quad (2.9)$$

After linearizing the nonlinear equations of motion given in equations 2.2 and 2.3, solving for the acceleration terms  $\ddot{\theta}$  and  $\ddot{\alpha}$ , and substituting the state given in , we obtain the following state-space matrices:

$$A = \frac{1}{J_T} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{1}{4}m_p L_p^2 L_r g & -\left(J_p + \frac{1}{4}m_p L_p^2\right) D_r & \frac{1}{2}m_p L_p L_r D_p \\ 0 & -\frac{1}{2}m_p L_p g (J_r + m_p L_r^2) & \frac{1}{2}m_p L_p L_r D_r & -\left(J_r + m_p L_r^2\right) D_p \end{bmatrix}$$

and

$$B = \frac{1}{J_T} \begin{bmatrix} 0 \\ 0 \\ J_p + \frac{1}{4}m_p L_p^2 \\ -\frac{1}{2}m_p L_p L_r \end{bmatrix}.$$

In the output equation, only the position of the servo and link angles are being measured. Based on this, the  $C$  and  $D$  matrices in the output equation are

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (2.10)$$

and

$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (2.11)$$

The 2 DOF Gantry does not have a rotary arm as described in Figure 2.1, it has a four-bar linkage. Thus the length of the 'arm' in terms of the link length  $L_b$  is defined as

$$L_r = \sqrt{L_b^2 + L_b^2} = \sqrt{2}L_b.$$

The velocities of the servo and pendulum angles can be computed in the digital controller, e.g., by taking the derivative and filtering the result through a high-pass filter.



## 2.2 Control

In Section 2.1, we found a linear state-state space model that represents a single rotary gantry system. This model is used to investigate the stability properties of the system in Section 2.2.2. In Section 2.2.3, the notion of controllability is introduced. Using the Linear Quadratic Regular algorithm, or LQR, is a common way to find the control gain and is discussed in Section 2.2.4. Lastly, Section 2.2.5 describes the state-feedback control used to control the servo position while minimizing link deflection.

### 2.2.1 Specifications

For our application, the suspended pendulum response performance can be assessed in terms of speed of response, minimum oscillation, and position accuracy. The time-domain specifications for both the servo and pendulum in the X- and Y-axes are:

**Specification 1:** Servo angle settling time:  $t_s \leq 1.5$  s.

**Specification 2:** Servo angle percentage overshoot:  $PO \leq 7.5$  %.

**Specification 3:** Servo angle steady-state error:  $e_{ss} \leq 0.5$  deg.

**Specification 4:** Maximum pendulum angle deflection:  $|\alpha|_{max} \leq 5$  deg.

**Specification 5:** Maximum control effort / voltage:  $|V_m| \leq 15$  V.

Thus when tracking a given reference, the response from the X and Y servo should have a settling time less than or equal to 1.5 seconds, an overshoot less than or equal to 7.5 %, and the steady-state response should have less than 0.5 degree error. The pendulum, in both the X and Y axes, must be kept within  $\pm 5$  degrees. Finally, the controller must not surpass the limits of the SRV02 motors and amplifiers.

### 2.2.2 Stability

The stability of a system can be determined from its poles ([8]):

- Stable systems have poles only in the left-hand plane.
- Unstable systems have at least one pole in the right-hand plane and/or poles of multiplicity greater than 1 on the imaginary axis.
- Marginally stable systems have one pole on the imaginary axis and the other poles in the left-hand plane.

The poles are the roots of the system's characteristic equation. From the state-space, the characteristic equation of the system can be found using

$$\det(sI - A) = 0 \quad (2.12)$$

where  $\det()$  is the determinant function,  $s$  is the Laplace operator, and  $I$  the identity matrix. These are the *eigenvalues* of the state-space matrix  $A$ .

### 2.2.3 Controllability

If the control input,  $u$ , of a system can take each state variable,  $x_i$  where  $i = 1 \dots n$ , from an initial state to a final state then the system is controllable, otherwise it is uncontrollable ([8]).

**Rank Test** The system is controllable if the rank of its controllability matrix

$$T = [B \ AB \ A^2B \ \dots \ A^nB] \quad (2.13)$$

equals the number of states in the system,

$$\text{rank}(T) = n. \quad (2.14)$$

### 2.2.4 Linear Quadratic Regular (LQR)

If  $(A,B)$  are controllable, then the Linear Quadratic Regular optimization method can be used to find a feedback control gain. Given the plant model in Equation 2.6, find a control input  $u$  that minimizes the cost function

$$J = \int_0^\infty x(t)' Q x(t) + u(t)' R u(t) dt, \quad (2.15)$$

where  $Q$  and  $R$  are the weighting matrices. The weighting matrices affect how LQR minimizes the function and are, essentially, tuning variables.

Given the control law  $u = -Kx$ , the state-space in Equation 2.6 becomes

$$\begin{aligned} \dot{x} &= Ax + B(-Kx) \\ &= (A - BK)x \end{aligned}$$

### 2.2.5 Feedback Control

The feedback control loop that in Figure 2.3 is designed to stabilize the servo to a desired position,  $\theta_d$ , while minimizing the deflection of the pendulum.

The reference state is defined

$$x_d = [\theta_d \ 0 \ 0 \ 0] \quad (2.16)$$

and the controller is

$$u = K(x_d - x). \quad (2.17)$$

Note that if  $x_d = 0$  then  $u = -Kx$ , which is the control used in the LQR algorithm.

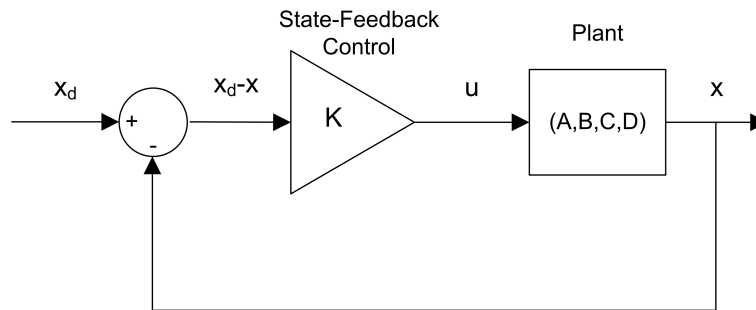


Figure 2.3: State-feedback control loop

# 3 LAB EXPERIMENTS

## 3.1 Simulation

In this section we will use the Simulink diagram shown in Figure 3.1 to simulate the closed-loop control of the *1 DOF Rotary Gantry* system. Recall in Section 2.1 the 2 DOF Gantry is modeled as two independent and identical gantry systems. We will only be examining the 1 DOF portion. The Rotary Gantry is simulated using a nonlinear model (it represents the actual system better than a linear model).

The Simulink model uses the state-feedback control described in Section 2.2.5. You can switch between *full-state* and *partial-state* feedback. In partial state-feedback, the user can examine how the system behaves when you cannot compensate for the swinging motions of the gantry. The feedback gain  $K$  is found using the Matlab LQR command (LQR is described briefly in Section 2.2.4). The goal is to find a gain that will satisfy the specifications given in Section 2.2.1.

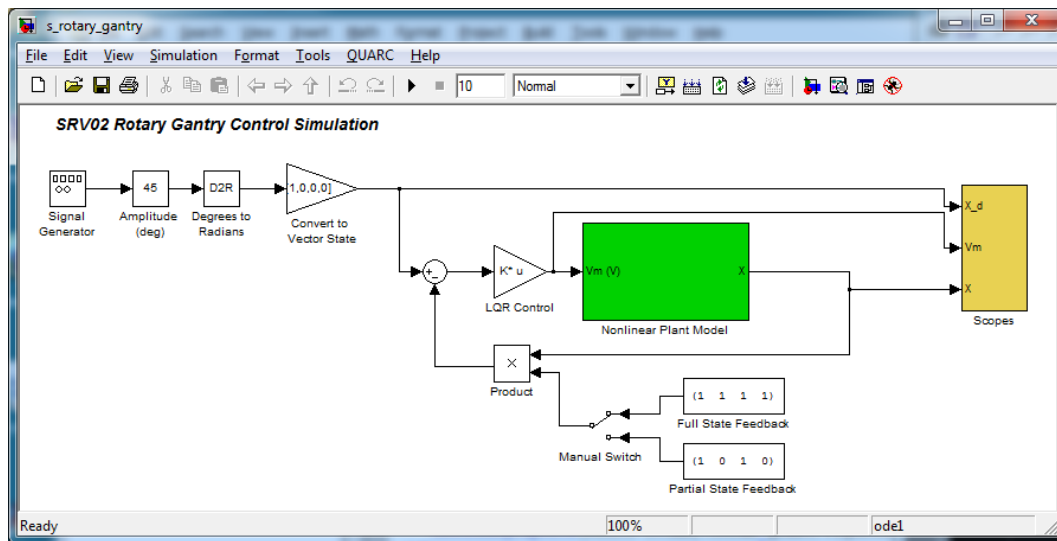


Figure 3.1: Simulink model used to simulate 1 DOF Rotary Gantry.

**IMPORTANT:** Before you can conduct these experiments, you need to make sure that the lab files are configured according to your setup. If they have not been configured already, then you need to go to Section 4 to configure the lab files first.

### 3.1.1 Procedure

Follow these steps to simulate the rotary gantry:

1. Make sure the LQR weighting matrices in `setup_2d_gantry.m` are to

$$Q = \begin{bmatrix} 250 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

and  $R = 1$ .

2. Run the script to generate the gain

$$K = \begin{bmatrix} 15.8 & -17.7 & 3.64 & 0.0080 \end{bmatrix}.$$

Remark: When tuning the LQR,  $Q(1,1)$  effects the servo proportional gain while  $Q(3,3)$  effects the servo derivative gain (which reduces the overshoot). Increasing  $Q(4,4)$  attenuates the motions of the pendulum.

3. To generate a 0.1 Hz square wave reference, ensure the *Signal Generator* is set to the following:
  - Signal type = *square*
  - Amplitude = 1
  - Frequency = 0.1 Hz
4. Set the *Amplitude (deg)* gain blocks to 10 to generate a step with an amplitude of 10 degrees (i.e., square wave goes between  $\pm 10$  which results in a step amplitude of 20).
5. Open the servo gear position scope,  $\theta_{s1}$  (rad), the pendulum angle scope,  $\alpha$  (deg), and the motor input voltage scope,  $V_m$  (V).
6. To simulate full-state feedback (FSF), set the Manual Switch to the *Full-State Feedback*.
7. Start the simulation. By default, the simulation runs for 15 seconds. The scopes should be displaying responses similar to Figure 3.2. Note that in the  $\theta_{s1}$  (rad) scopes, the yellow trace is the setpoint position while the purple trace is the simulated position.

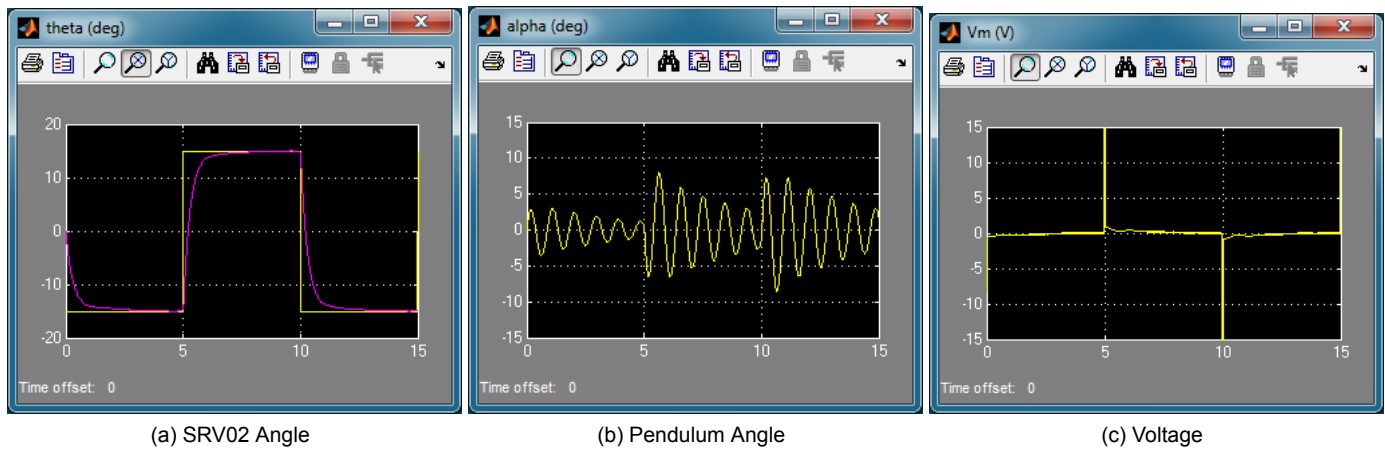


Figure 3.2: Simulated closed-loop response.

### 3.1.2 Full-State Feedback Analysis

The full-state feedback response is shown in Figure 3.3. You can generate this figure by running the `plot_response_2d_gantry_sim.m` script.

From the response shown in Figure 3.3, it is clear that the steady-state error is zero, thus

$$e_{ss} = 0$$

The settling time, percent overshoot, and maximum deflection of the response are

$$\begin{aligned} t_s &= 1.14 \text{ sec} \\ PO &= 4.4 \% \\ |\alpha|_{max} &= 4.42 \text{ deg} \end{aligned}$$

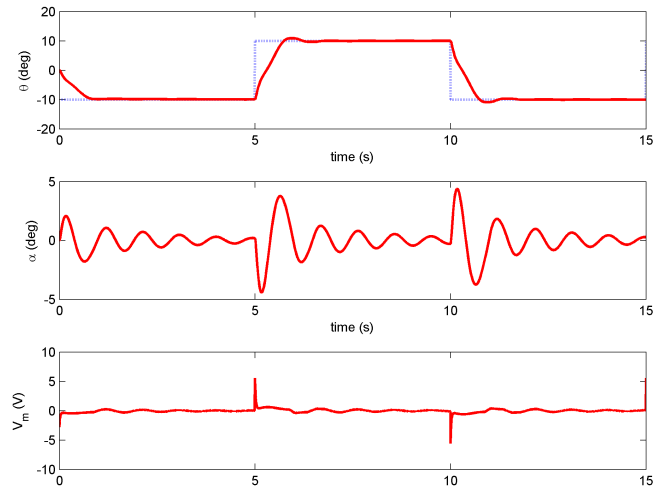


Figure 3.3: Simulated 1 DOF Gantry full-state feedback response.

The simulated response satisfies the specifications given in Section 2.2.1 while maintaining a motor input voltage less than 15 V, i.e., the motor is not saturated. The specifications are automatically found when the `plot_response_2d_gantry_sim.m` script is ran.

**Generating the Matlab figure:** After each simulation run, each scope automatically saves their response to a variable in the **Matlab®** workspace. The *theta (deg)* scope saves its response to the variable called `data_theta`, the *alpha (deg)* scope saves its data to the `data_alpha` variable, and the *Vm (V)* scope saves its plot to the `data_vm` variable. See the `plot_response_2d_gantry_sim.m` script to see how they are used for plotting.

### 3.1.3 Partial-State Feedback Analysis

Using the same gain  $K$ , run the simulation when the *Manual Switch* is set to *Partial-State Feedback* mode. After running `plot_response_2d_gantry_sim.m` script you obtain the response shown in Figure 3.4.

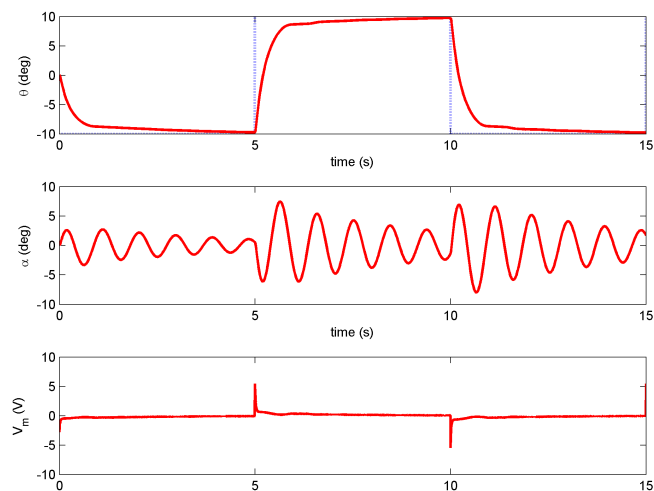


Figure 3.4: Simulated 1 DOF Gantry partial-state feedback response.

As shown the pendulum (i.e., crane) tends to oscillate much more as the servo rotates to track its desired position. The servo response is also slowed down. The steady-state error, settling time, percent overshoot, and maximum deflection of the simulated partial-state feedback response are

$$\begin{aligned} e_{ss} &= 0.21 \text{ deg} \\ t_s &= 2.7 \text{ sec} \\ PO &= 0 \% \\ |\alpha|_{max} &= 8.0 \text{ deg} \end{aligned}$$

## 3.2 Implementation

The q\_2d\_gantry Simulink diagram shown in Figure 3.5 is used to perform the gantry control on the 2 DOF Gantry. The 2 DOF Gantry subsystem contains QUARC® blocks that interface with the DC motor and sensors of the 2 DOF Gantry system.

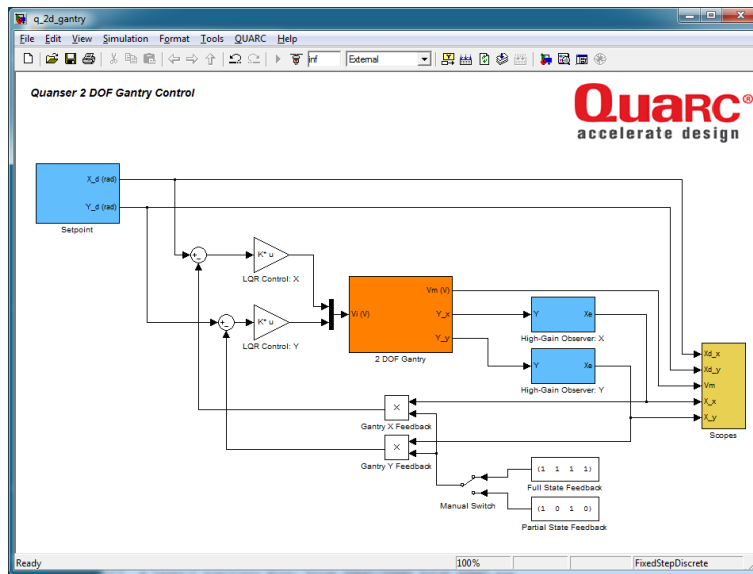


Figure 3.5: Simulink model used with QUARC® to run controller on the 2 DOF Gantry.

**IMPORTANT:** Before you can conduct these experiments, you need to make sure that the lab files are configured according to your setup. If they have not been configured already, then you need to go to Section 4 to configure the lab files first.

### 3.2.1 Procedure

Follow this procedure:

1. Run the setup\_2d\_gantry.m script using the LQR weighting matrices that you used in the simulation in Section 3.1.
2. To generate a step reference, go to the *Setpoint* subsystem ensure the each *Signal Generator* is set to the following:
  - Signal type = *square*
  - Amplitude = 1
  - Frequency = 0.5 Hz

3. In the *Setpoint* subsystem, set both *Amplitude (deg)* gain blocks to 10 to generate a step with an amplitude of 10 degrees (i.e., square wave goes between  $\pm 10$  which results in a step amplitude of 20).
4. Open the load shaft position scope,  $\theta_I$  (rad), and the motor input voltage scope,  $V_m$  (V).
5. Open the servo X gear position scope,  $\theta_{x\text{ (deg)}}$ , the pendulum X angle scope,  $\alpha_x$  (deg), servo Y gear position scope,  $\theta_{y\text{ (deg)}}$ , the pendulum Y angle scope,  $\alpha_y$  (deg), and the motor input voltage scope,  $V_m$  (V).
6. In the Simulink diagram, go to QUARC | Build.
7. **Make sure the system is in the HOME position**, as shown in Section 2.1.1 (see [7] for more information).
8. Click on QUARC | Start to run the controller. The servos should begin rotation back-and-forth and the gantry should be moving in a square-like fashion. The scopes should be displaying responses similar to Figure 3.6. Note that in the  $\theta_{x\text{ (deg)}}$  and  $\theta_{y\text{ (deg)}}$  scopes, the yellow trace is the setpoint position while the purple trace is the measured position.

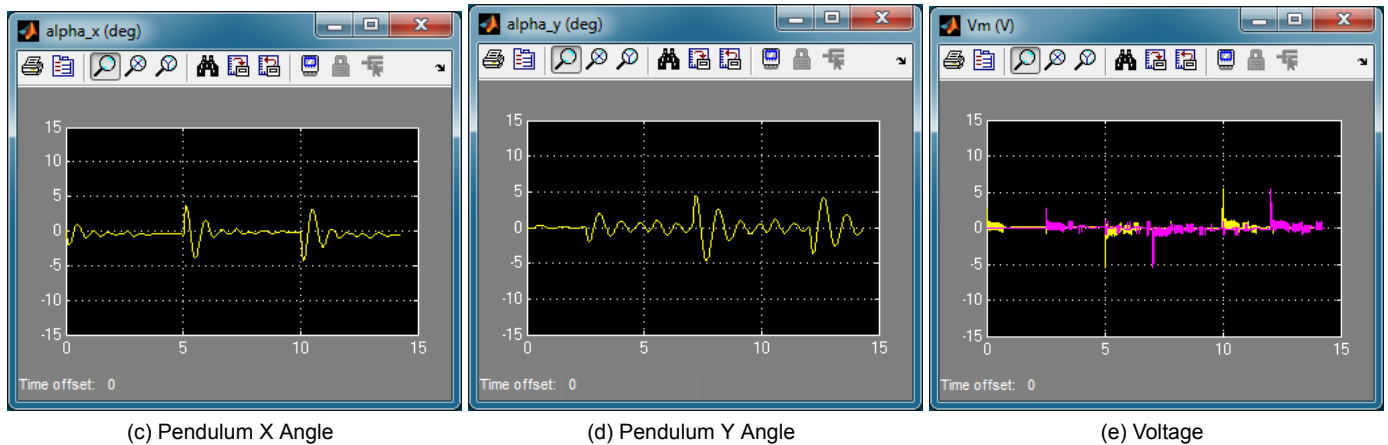
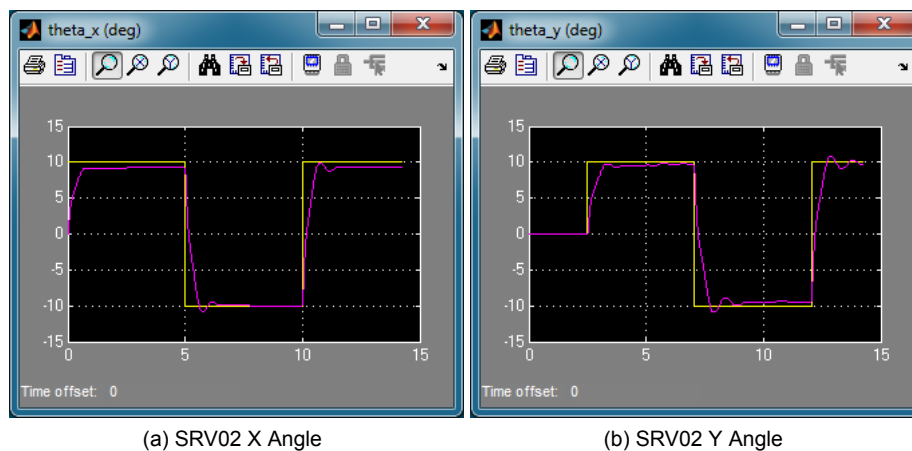
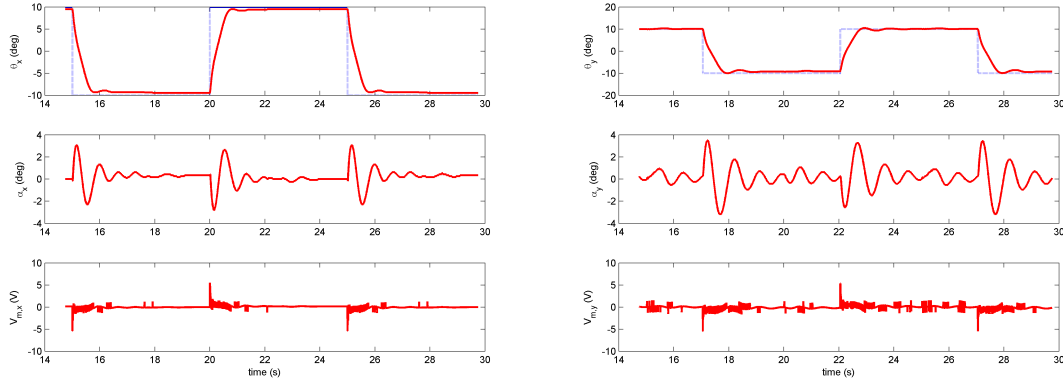


Figure 3.6: Typical response when running control on 2 DOF Gantry system

### 3.2.2 Full-State Feedback Analysis

The closed-loop position response is shown in Figure 3.7. You can generate this using the `plot_response_2d_gantry.m` script.

For the x-axis shown in Figure 3.7, the steady-state error, settling time, percentage overshoot, and maximum pen-



(a) X-Axis

(b) Y-Axis

Figure 3.7: 2 DOF Gantry full-state feedback response

dulum deflection are:

$$\begin{aligned} e_{ss} &= 0.51 \text{ deg}, \\ t_s &= 0.68 \text{ sec}, \\ PO &= 0\%, \text{ and} \\ |\alpha|_{max} &= 3.1 \text{ deg} \end{aligned}$$

For the y-axis the measured specifications are:

$$\begin{aligned} e_{ss} &= -0.020 \text{ deg}, \\ t_s &= 0.82 \text{ sec}, \\ PO &= 2.3\%, \text{ and} \\ |\alpha|_{max} &= 3.5 \text{ deg}. \end{aligned}$$

For the most part, the specifications given in Section 2.2.1 are satisfied for both axes while maintaining a motor input voltage less than 15 V, i.e., the motor is not saturated. The steady-state error of the x-axis marginally exceeds the limit. With further tuning of the LQR, this could be improved. The specifications are automatically found using when the `plot_response_2d_gantry.m` script is ran.

### 3.2.3 Partial-State Feedback Analysis

With the same gain  $K$ , run the `q_2d_gantry` controller when the *Manual Switch* is set to *Partial-State Feedback* mode. After running `plot_response_2d_gantry.m` script you obtain the response shown in Figure 3.8.

As in the simulation, the pendulum tends to swing more as the servos track their desired angles. The specifications measured on the partial-state feedback response captured in Figure 3.8 for the x-axis are:

$$\begin{aligned} e_{ss} &= 0.51 \text{ deg}, \\ t_s &= 2.14 \text{ sec}, \\ PO &= 0\%, \text{ and} \\ |\alpha|_{max} &= 5 \text{ deg} \end{aligned}$$



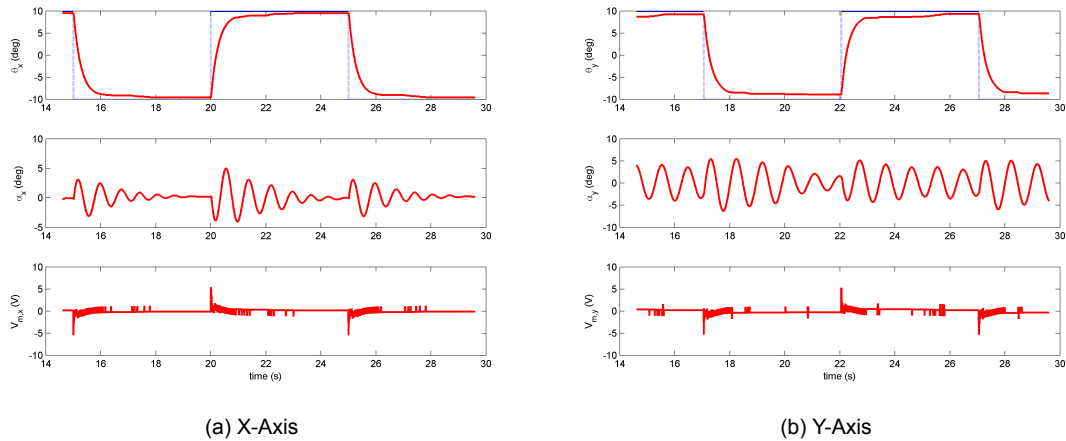


Figure 3.8: 2 DOF Gantry partial-state feedback response

and for the y-axis:

$$\begin{aligned}
 e_{ss} &= 0.68 \text{ deg}, \\
 t_s &= 3.34 \text{ sec}, \\
 PO &= 0\%, \text{ and} \\
 |\alpha|_{max} &= 6.3 \text{ deg}.
 \end{aligned}$$

The settling time on both servos has increased, i.e., servo response has slowed down, and no longer satisfies the specifications given in Section 2.2.1. The pendulum swing along the y-axis exceed the specification as well.

# 4 SYSTEM REQUIREMENTS

## Required Software

- Microsoft Visual Studio (MS VS)
- **Matlab®** with **Simulink®**, Real-Time Workshop, and the Control System Toolbox
- **QUARC®**

See the **QUARC®** software compatibility chart in [4] to see what versions of MS VS and Matlab are compatible with your version of QUARC and for what OS.

## Required Hardware

- Data acquisition (DAQ) device **with four encoder inputs** and that is compatible with **QUARC®**. This includes Quanser DAQ boards such as Q8-USB, QPID, and QPIDe and some National Instruments DAQ devices. For a full listing of compliant DAQ cards, see Reference [1].
- 2x Quanser SRV02-ET rotary servos.
- Quanser 2 DOF Robot (four bar linkage attached to the SRV02 units).
- Quanser SRV02 2 DOF Gantry (attached to end-effector on 2 DOF Robot).
- Quanser VoltPAQ-X1 power amplifier, or equivalent.

## Before Starting Lab

Before you begin this laboratory make sure:

- **QUARC®** is installed on your PC, as described in [2].
- DAQ device has been successfully tested (e.g., using the test software in the Quick Start Guide or the *QUARC Analog Loopback Demo*).
- SRV02 2 DOF Gantry and amplifier are connected to your DAQ board as described its User Manual [6].

## 4.1 Overview of Files

File Name	Description
2 DOF Gantry User Manual.pdf	This manual describes the hardware of the 2 DOF Gantry system and explains how to setup and wire the system for the experiments.
2 DOF Gantry Laboratory Guide.pdf	This document demonstrates how to obtain the linear state-space model of the system, simulate the closed-loop system, and implement controllers on the 2 DOF Gantry plant using <b>QUARC®</b> .
setup_2d_gantry.m	The main Matlab script that sets the SRV02 motor and sensor parameters, the SRV02 configuration-dependent model parameters, and the 2 DOF Gantry sensor parameters. Run this file only to setup the laboratory.
config_srv02.m	Returns the configuration-based SRV02 model specifications $R_m$ , $k_t$ , $k_m$ , $K_g$ , $\eta_{a_g}$ , $B_{eq}$ , $J_{eq}$ , and $\eta_{a_m}$ , the sensor calibration constants $K_{POT}$ , $K_{ENC}$ , and $K_{TACH}$ , and the amplifier limits $V_{MAX\_AMP}$ and $I_{MAX\_AMP}$ .
config_2d_robot.m	Returns the 2 DOF Robot related model parameters.
config_2d_gantry.m	Returns the 2 DOF Gantry related model parameters.
calc_conversion_constants.m	Returns various conversions factors.
s_rotary_gantry.mdl	Simulink file that simulates the closed-loop control of a Rotary Gantry system using state-feedback control.
q_2d_gantry.mdl	Simulink file that implements the state-feedback control on the 2 DOF Gantry system using <b>QUARC®</b> .
2d_robot.mws	Maple worksheet used to develop the model for the 2 DOF Gantry experiment. Waterloo Maple 9, or a later release, is required to open, modify, and execute this file.
2d_robot.html	HTML presentation of the Maple Worksheet. It allows users to view the content of the Maple file without having Maple 9 installed. No modifications to the equations can be performed when in this format.
plot_response_2d_gantry_sim.m	Plots the response found in the variables <code>data_theta</code> , <code>data_alpha</code> , and <code>data_vm</code> in the Matlab workspace and measures the corresponding peak time, settling time, percent overshoot, and steady-state error. These variables are saved from the Simulink diagrams <code>s_rotary_gantry.mdl</code> .
plot_response_2d_gantry.m	Plots the response found in the variables <code>data_theta_x</code> , <code>data_theta_y</code> , <code>data_alpha_x</code> , <code>data_alpha_y</code> , and <code>data_vm</code> in the Matlab workspace and measures the corresponding peak time, settling time, percent overshoot, and steady-state error. These variables are saved from the Simulink diagrams <code>q_2d_gantry.mdl</code> .
meas_step_rsp_specs.m	Function that measures the peak time, settling time, steady-state error, and percent overshoot of a given step response.

Table 4.1: Files supplied with the 2 DOF Gantry

## 4.2 Setup for Simulation

Before beginning the in-lab procedure outlined in Section 3.1, the `s_rotary_gantry` Simulink diagram and the `setup_2d_gantry.m` script must be configured.

Follow these steps:

1. Load the Matlab software.
2. Browse through the *Current Directory* window in Matlab and find the folder that contains the file *setup\_2d\_gantry.m*.
3. Open the *setup\_2d\_gantry.m* script.
4. **Configure *setup\_2d\_gantry.m* script:** When used with the 2 DOF Gantry, the SRV02 has no load (i.e., no disc or bar) and has to be in the high-gear configuration. Make sure the script is setup to match this setup:
  - EXT\_GEAR\_CONFIG to 'HIGH'
  - LOAD\_TYPE to 'NONE'
  - K\_AMP to 1 (unless your amplifier gain is different)
  - AMP\_TYPE to your amplifier type (e.g., VoltPAQ).
  - Ensure other parameters such as ENCODER\_TYPE, TACH\_OPTION, and VMAX\_DAC match your system configuration.
5. Run *setup\_2d\_gantry.m* to setup the Matlab workspace.
6. Open the *s\_rotary\_gantry.mdl* Simulink diagram, shown in Figure 3.1.

## 4.3 Setup for Running on 2 DOF Gantry

Before performing the in-lab exercises in Section 3.2, the *q\_2d\_gantry* Simulink diagram and the *setup\_2d\_gantry.m* script must be configured.

Follow these steps to get the system ready for this lab:

1. Setup the SRV02 with the 2 DOF Gantry module as detailed in the 2 DOF Gantry User Manual [6].
2. **Make sure the 2 DOF Gantry is in the HOME position**, as depicted in Section 2.1.1. For more information about the HOME position, go to the 2 DOF Gantry User Manual.
3. Configure and run *setup\_2d\_gantry.m* as explained in Section 4.2.
4. Open the *q\_2d\_gantry.mdl* Simulink diagram, shown in Figure 3.5.
5. **Configure DAQ:** Ensure the HIL Initialize block in the *2 DOF Gantry* subsystem is configured for the DAQ device that is installed in your system. See Reference [1] for more information on configuring the HIL Initialize block.

# REFERENCES

- [1] Quanser Inc. *QUARC User Manual*.
- [2] Quanser Inc. *QUARC Installation Guide*, 2009.
- [3] Quanser Inc. *SRV02 User Manual*, 2009.
- [4] Quanser Inc. *QUARC Compatibility Table*, 2010.
- [5] Quanser Inc. *SRV02 lab manual*. 2011.
- [6] Quanser Inc. *2 DOF Gantry User Manual*, 2012.
- [7] Quanser Inc. *2 DOF Robot User Manual*, 2012.
- [8] Norman S. Nise. *Control Systems Engineering*. John Wiley & Sons, Inc., 2008.

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