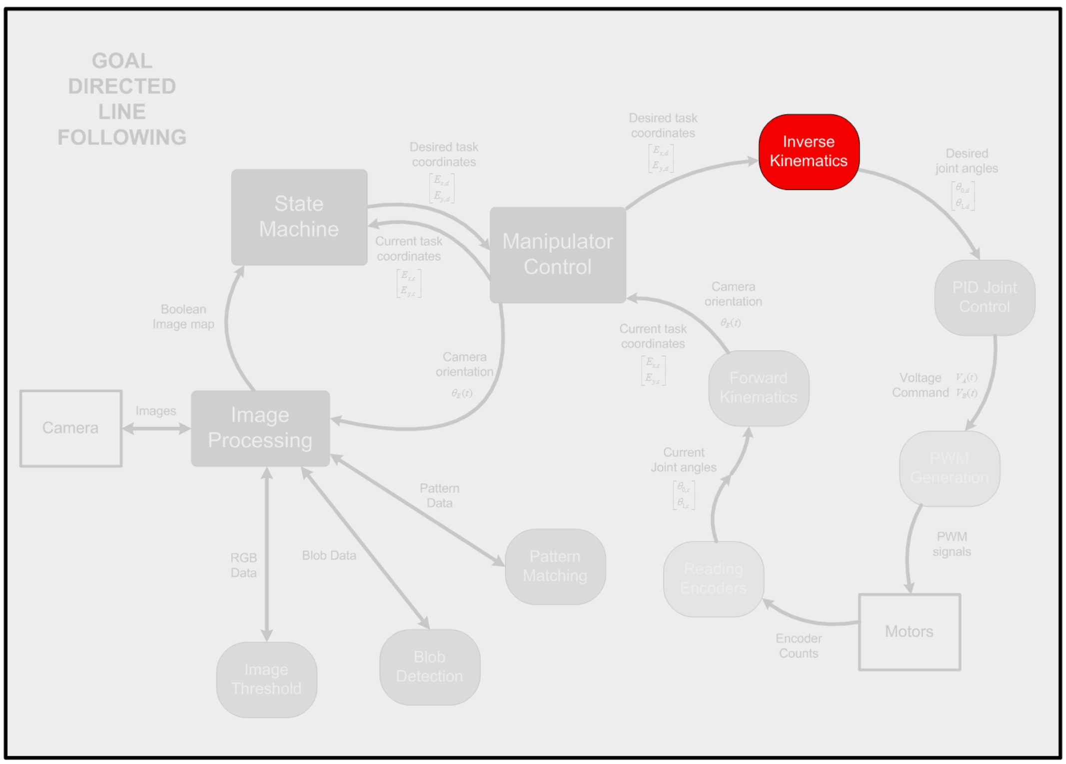
Inverse Kinematics

Topics Covered

- Obtaining the Inverse Kinematics equations using a geometric approach.
- Using LabVIEW™ to develop an Inverse Kinematics VI.
- Model validation using encoder information.

Prerequisites

- The QNET Mechatronic Systems is set up according to the Quick Start Guide.



1 Background

For any robotic system, inverse kinematics involves determining the manipulator state $\vec{\theta}$, given the end-effector position $\vec{E}$. For the QNET Mechatronic Systems shown in Figure 1.1, this corresponds to

Find,

$$\vec{\theta} = \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} \tag{1.1}$$

Given,

$$\vec{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} \tag{1.2}$$

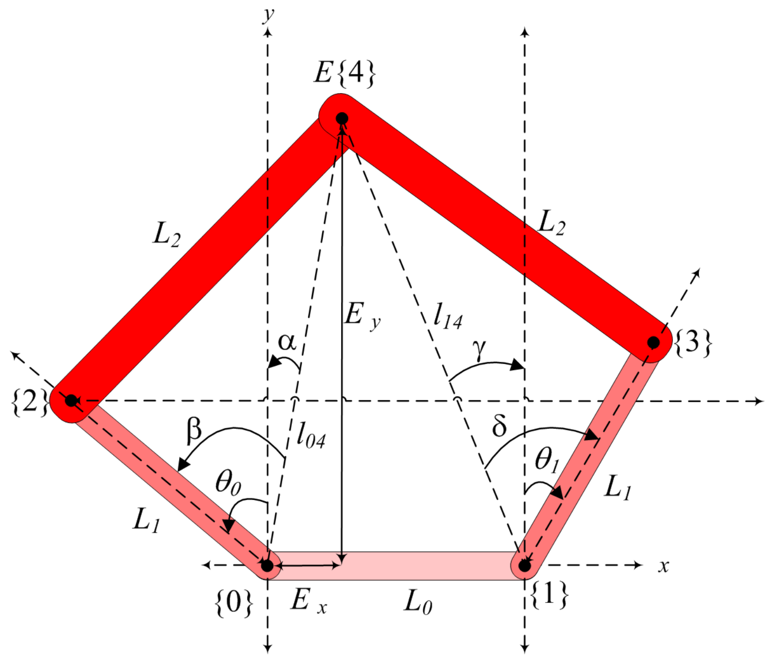


Figure 1.1: QNET Mechatronic Systems kinematic diagram

The inverse kinematics equations can be developed using a geometric approach, as follows:

$$\begin{aligned}
l_{04} &= \sqrt{(E_x)^2 + (E_y)^2} \\
\alpha &= 90 - \tan^{-1}\left(\frac{E_y}{E_x}\right) \\
\beta &= \cos^{-1}\left(\frac{-{L_2}^2 + {L_1}^2 + l_{04}^2}{2L_1 l_{04}}\right) \\
\theta_0 &= \beta - \alpha
\end{aligned} \tag{1.3}$$

followed by,

$$
\begin{aligned}
l_{14} &= \sqrt{(L_0 - E_x)^2 + (E_y)^2} \\
\gamma &= 90 - \tan^{-1}\left(\frac{E_y}{L_0 - E_x}\right) \\
\delta &= \cos^{-1}\left(\frac{-{L_2}^2 + {L_1}^2 + l_{14}^2}{2L_1 l_{14}}\right) \\
\theta_1 &= \delta - \gamma
\end{aligned} \tag{1.4}
$$

Note that using the inverse cosine works for this situation, because β and δ are in the open interval (0,π). The link parameters for the QNET Mechatronic Systems are provided in Table 1.1.

Symbol	Description	Value
L_0	Fixed length between two motors	7.6 cm
L_1	Length of links $\{0\}$-$\{2\}$ and $\{1\}$-$\{3\}$	8.4 cm
L_2	Length of links $\{2\}$-$\{4\}$ and $\{3\}$-$\{4\}$	12.6 cm

Table 1.1: QNET Mechatronic Systems link parameters

2 In-Lab Exercises

1. Open `Mechatronic Systems.lvproj`, and under `Quanser ELVIS RIO | Subsystems`, open `Inverse Kinematics.vi`. Open `Inv Kin.vi`. Complete the model according to that shown in Figure 2.1. The code is similar to Equation 1.3 and Equation 1.4. Test the VI with the following values for $\vec{E}$ and comment on the results.

$$\vec{E} = \begin{bmatrix} 3.8 \\ 20.4 \end{bmatrix}, \begin{bmatrix} -4.4 \\ 11.9 \end{bmatrix}, \begin{bmatrix} 12.1 \\ 11.9 \end{bmatrix} \tag{2.1}$$

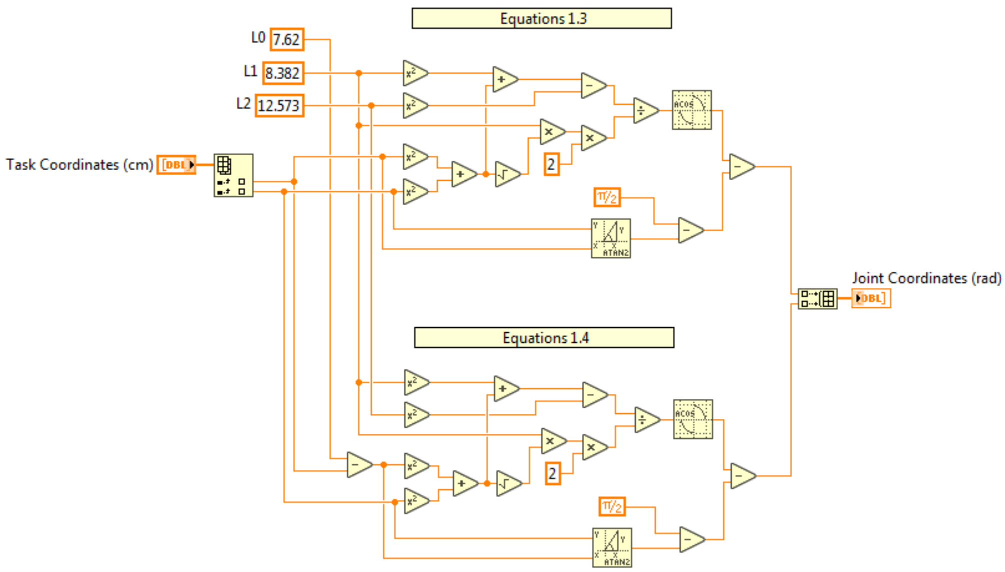


Figure 2.1: Inverse Kinematics formulation

2. Close `Inv Kin.vi` and run `Inverse Kinematics.vi`. Once the `Calibration` bar is full, move the manipulator around manually to the symbols in Table 2.1. Enter the coordinates of the corresponding symbol in the position control and compare the actual motor angles with the values from `Inv Kin.vi`. Comment on the results.

   Note: If this is the first time any of the VIs is being run, calibration might take up to 10 s.

Symbol	E_x	E_y
Star	6.3 cm	12.3 cm
Plus	-1.3 cm	11.4 cm
X	1.7 cm	18.5 cm

Table 2.1: QNET Mechatronic Systems Symbol Locations

Quanser Inc.
119 Spy Court
Markham, Ontario
L3R 5H6
Canada
info@quanser.com
Phone: 1-905-940-3575
Fax: 1-905-940-3576

Printed in Markham, Ontario.

For more information on the solutions Quanser Inc. offers, please visit the web site at:
http://www.quanser.com