



STUDENT WORKBOOK

SRV02 Base Unit Experiment For Matlab®/Simulink® Users

Standardized for ABET Evaluation Criteria

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ROTARY SERVO BASE UNIT MODELING

The objective of this experiment is to find a transfer function that describes the rotary motion of the Rotary Servo Base Unit load shaft. The dynamic model is derived analytically from classical mechanics principles and using experimental methods.

Topics Covered

- Deriving the dynamics equation and transfer function for the Rotary Servo Base Unit servo plant using the first-principles.
- Obtaining the Rotary Servo Base Unit transfer function using a frequency response experiment.
- Obtaining the Rotary Servo Base Unit transfer function using a bump test.
- Tuning the obtained transfer function and validating it with the actual system response.

Prerequisites

- System has been setup and tested by going through the Rotary Servo Base Unit Quick Start Guide.
- Familiar with Transfer function fundamentals, e.g. obtaining a transfer function from a differential equation.
- Familiar with **MATLAB®** and **SIMULINK®** fundamentals.
- Integration laboratory experiment to get familiar with using **QUARC®** with the Rotary Servo Base Unit..



Caution

Make sure the system has been setup and tested by going through the Rotary Servo Base Unit Quick Start Guide before starting this experiment!

1 Background

The angular speed of the Rotary Servo Base Unit load shaft with respect to the input motor voltage can be described by the following first-order transfer function

$$\frac{\Omega_l(s)}{V_m(s)} = \frac{K}{\tau s + 1} \quad (1.1)$$

where $\Omega_l(s)$ is the Laplace transform of the load shaft speed $\omega_l(t)$, $V_m(s)$ is the Laplace transform of motor input voltage $v_m(t)$, K is the steady-state gain, τ is the time constant, and s is the Laplace operator.

The Rotary Servo Base Unit transfer function model is derived analytically in Section 1.1 and its K and τ parameters are evaluated. These are known as the nominal model parameter values. The model parameters can also be found experimentally. Sections 1.2.1 and 1.2.2 describe how to use the frequency response and bump-test methods to find K and τ . These methods are useful when the dynamics of a system are not known, for example in a more complex system. After the lab experiments, the experimental model parameters are compared with the nominal values.

1.1 Modeling Using First-Principles

1.1.1 Electrical Equations

The DC motor armature circuit schematic and gear train is illustrated in Figure Figure 1.1. As specified in Rotary Servo Base Unit User Manual, recall that R_m is the motor resistance, L_m is the inductance, and k_m is the back-emf constant.

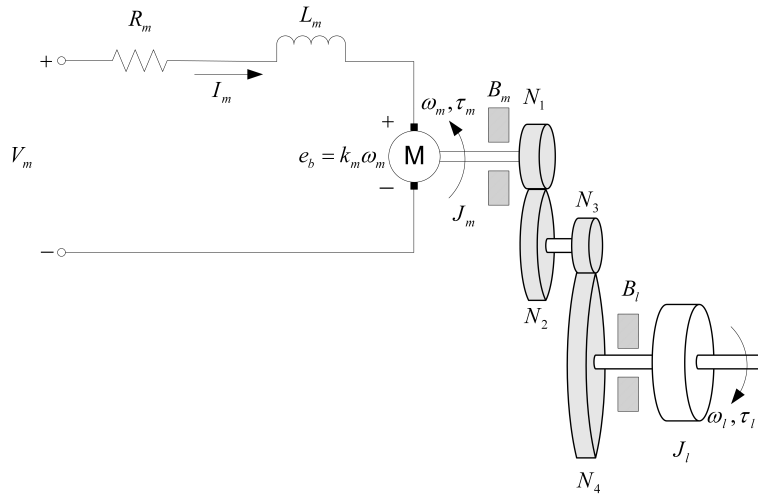


Figure 1.1: Rotary Servo Base Unit DC motor armature circuit and gear train

The back-emf (electromotive) voltage $e_b(t)$ depends on the speed of the motor shaft, ω_m , and the back-emf constant of the motor, k_m . It opposes the current flow. The back emf voltage is given by:

$$e_b(t) = k_m \omega_m(t) \quad (1.2)$$

Using Kirchoff's Voltage Law, we can write the following equation:

$$V_m(t) - R_m I_m(t) - L_m \frac{dI_m(t)}{dt} - k_m \omega_m(t) = 0 \quad (1.3)$$

Since the motor inductance L_m is much less than its resistance, it can be ignored. Then, the equation becomes

$$V_m(t) - R_m I_m(t) - k_m \omega_m(t) = 0 \quad (1.4)$$

Solving for $I_m(t)$, the motor current can be found as:

$$I_m(t) = \frac{V_m(t) - k_m \omega_m(t)}{R_m} \quad (1.5)$$

1.1.2 Mechanical Equations

In this section the equation of motion describing the speed of the load shaft, ω_l , with respect to the applied motor torque, τ_m , is developed.

Since the Rotary Servo Base Unit is a one degree-of-freedom rotary system, Newton's Second Law of Motion can be written as:

$$J \cdot \alpha = \tau \quad (1.6)$$

where J is the moment of inertia of the body (about its center of mass), α is the angular acceleration of the system, and τ is the sum of the torques being applied to the body. As illustrated in Figure 1.1, the Rotary Servo Base Unit gear train along with the viscous friction acting on the motor shaft, B_m , and the load shaft B_l are considered. The load equation of motion is

$$J_l \frac{d\omega_l(t)}{dt} + B_l \omega_l(t) = \tau_l(t) \quad (1.7)$$

where J_l is the moment of inertia of the load and τ_l is the total torque applied on the load. The load inertia includes the inertia from the gear train and from any external loads attached, e.g. disc or bar. The motor shaft equation is expressed as:

$$J_m \frac{d\omega_m(t)}{dt} + B_m \omega_m(t) + \tau_{ml}(t) = \tau_m(t) \quad (1.8)$$

where J_m is the motor shaft moment of inertia and τ_{ml} is the resulting torque acting on the motor shaft from the load torque. The torque at the load shaft from an applied motor torque can be written as:

$$\tau_l(t) = \eta_g K_g \tau_m(t) \quad (1.9)$$

where K_g is the gear ratio and η_g is the gearbox efficiency. The planetary gearbox that is directly mounted on the Rotary Servo Base Unit motor (see Rotary Servo Base Unit User Manual for more details) is represented by the N_1 and N_2 gears in Figure 1.1 and has a gear ratio of

$$K_{gi} = \frac{N_2}{N_1} \quad (1.10)$$

This is the *internal* gear box ratio. The motor gear N_3 and the load gear N_4 are directly meshed together and are visible from the outside. These gears comprise the *external* gear box which has an associated gear ratio of

$$K_{ge} = \frac{N_4}{N_3} \quad (1.11)$$

The gear ratio of the Rotary Servo Base Unit gear train is then given by:

$$K_g = K_{ge} K_{gi} \quad (1.12)$$

Thus, the torque seen at the motor shaft through the gears can be expressed as:

$$\tau_{ml}(t) = \frac{\tau_l(t)}{\eta_g K_g} \quad (1.13)$$

Intuitively, the motor shaft must rotate K_g times for the output shaft to rotate one revolution.

$$\theta_m(t) = K_g \theta_l(t) \quad (1.14)$$

We can find the relationship between the angular speed of the motor shaft, ω_m , and the angular speed of the load shaft, ω_l by taking the time derivative:

$$\omega_m(t) = K_g \omega_l(t) \quad (1.15)$$

To find the differential equation that describes the motion of the load shaft with respect to an applied motor torque substitute (Equation 1.13), (Equation 1.15) and (Equation 1.7) into (Equation 1.8) to get the following:

$$J_m K_g \frac{d\omega_l(t)}{dt} + B_m K_g \omega_l(t) + \frac{J_l \left(\frac{d\omega_l(t)}{dt} \right) + B_l \omega_l(t)}{\eta_g K_g} = \tau_m(t) \quad (1.16)$$

Collecting the coefficients in terms of the load shaft velocity and acceleration gives

$$(\eta_g K_g^2 J_m + J_l) \frac{d\omega_l(t)}{dt} + (\eta_g K_g^2 B_m + B_l) \omega_l(t) = \eta_g K_g \tau_m(t) \quad (1.17)$$

Defining the following terms:

$$J_{eq} = \eta_g K_g^2 J_m + J_l \quad (1.18)$$

$$B_{eq} = \eta_g K_g^2 B_m + B_l \quad (1.19)$$

simplifies the equation as:

$$J_{eq} \frac{d\omega_l(t)}{dt} + B_{eq} \omega_l(t) = \eta_g K_g \tau_m(t) \quad (1.20)$$

1.1.3 Combining the Electrical and Mechanical Equations

In this section the electrical equation derived in Section 1.1.1 and the mechanical equation found in Section 1.1.2 are brought together to get an expression that represents the load shaft speed in terms of the applied motor voltage.

The motor torque is proportional to the voltage applied and is described as

$$\tau_m(t) = \eta_m k_t I_m(t) \quad (1.21)$$

where k_t is the current-torque constant ($N.m/A$), η_m is the motor efficiency, and I_m is the armature current. See Rotary Servo Base Unit User Manual for more details on the Rotary Servo Base Unit motor specifications.

We can express the motor torque with respect to the input voltage $V_m(t)$ and load shaft speed $\omega_l(t)$ by substituting the motor armature current given by equation Equation 1.5 in Section 1.1.1, into the current-torque relationship given in equation Equation 1.21:

$$\tau_m(t) = \frac{\eta_m k_t (V_m(t) - k_m \omega_m(t))}{R_m} \quad (1.22)$$

To express this in terms of V_m and ω_l , insert the motor-load shaft speed equation Equation 1.15, into Equation 1.21 to get:

$$\tau_m(t) = \frac{\eta_m k_t (V_m(t) - k_m K_g \omega_l(t))}{R_m} \quad (1.23)$$

If we substitute (Equation 1.23) into (Equation 1.20), we get:

$$J_{eq} \left(\frac{d}{dt} \omega_l(t) \right) + B_{eq} \omega_l(t) = \frac{\eta_g K_g \eta_m k_t (V_m(t) - k_m K_g \omega_l(t))}{R_m} \quad (1.24)$$

After collecting the terms, the equation becomes

$$\left(\frac{d}{dt} \omega_l(t) \right) J_{eq} + \left(\frac{k_m \eta_g K_g^2 \eta_m k_t}{R_m} + B_{eq} \right) \omega_l(t) = \frac{\eta_g K_g \eta_m k_t V_m(t)}{R_m} \quad (1.25)$$

This equation can be re-written as:

$$\left(\frac{d}{dt} \omega_l(t) \right) J_{eq} + B_{eq,v} \omega_l(t) = A_m V_m(t) \quad (1.26)$$

where the equivalent damping term is given by:

$$B_{eq,v} = \frac{\eta_g K_g^2 \eta_m k_t k_m + B_{eq} R_m}{R_m} \quad (1.27)$$

and the actuator gain equals

$$A_m = \frac{\eta_g K_g \eta_m k_t}{R_m} \quad (1.28)$$

1.2 Modeling Using Experiments

In Section 1.1 you learned how the system model can be derived from the first-principles. A linear model of a system can also be determined purely experimentally. The main idea is to experimentally observe how a system reacts to different inputs and change structure and parameters of a model until a reasonable fit is obtained. The inputs can be chosen in many different ways and there are a large variety of methods. In Sections 1.2.1 and 1.2.2, two methods of modeling the Rotary Servo Base Unit are outlined: (1) frequency response and, (2) bump test.

1.2.1 Frequency Response

In Figure 1.2, the response of a typical first-order time-invariant system to a sine wave input is shown. As it can be seen from the figure, the input signal (u) is a sine wave with a fixed amplitude and frequency. The resulting output (y) is also a sinusoid with the *same* frequency but with a different amplitude. By varying the frequency of the input sine wave and observing the resulting outputs, a Bode plot of the system can be obtained as shown in Figure 1.3.

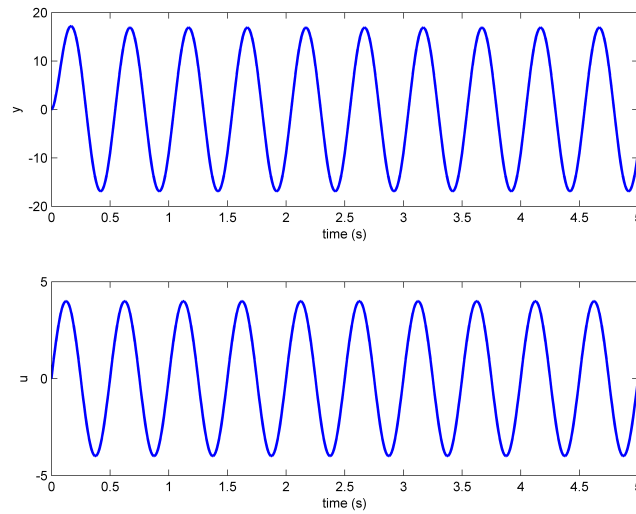


Figure 1.2: Typical frequency response

The Bode plot can then be used to find the steady-state gain, i.e. the DC gain, and the time constant of the system. The cutoff frequency, ω_c , shown in Figure 1.3 is defined as the frequency where the gain is 3 dB less than the maximum gain (i.e. the DC gain). When working in the linear non-decibel range, the 3 dB frequency is defined as the frequency where the gain is $\frac{1}{\sqrt{2}}$, or about 0.707, of the maximum gain. The cutoff frequency is also known as the bandwidth of the system which represents how fast the system responds to a given input.

The magnitude of the frequency response of the Rotary Servo Base Unit plant transfer function given in equation Equation 1.1 is defined as:

$$|G_{wl,v}(w)| = \left| \frac{\Omega_l(\omega j)}{V_m(\omega j)} \right| \quad (1.29)$$

where ω is the frequency of the motor input voltage signal V_m . We know that the transfer function of the system has the generic first-order system form given in Equation Equation 1.1. By substituting $s = j\omega$ in this equation, we can

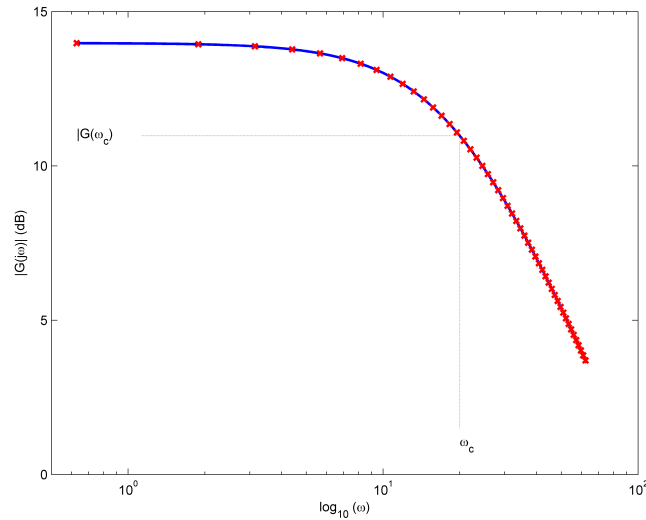


Figure 1.3: Magnitude Bode plot

find the frequency response of the system as:

$$\frac{\Omega_l(\omega j)}{V_m(\omega j)} = \frac{K}{\tau\omega j + 1} \quad (1.30)$$

Then, the magnitude of it equals

$$|G_{wl,v}(\omega)| = \frac{K}{\sqrt{1 + \tau^2 \omega^2}} \quad (1.31)$$

Let's call the frequency response model parameters $K_{e,f}$ and $\tau_{e,f}$ to differentiate them from the nominal model parameters, K and τ , used previously. The steady-state gain or the DC gain (i.e. gain at zero frequency) of the model is:

$$K_{e,f} = |G_{wl,v}(0)| \quad (1.32)$$

1.2.2 Bump Test

The bump test is a simple test based on the step response of a stable system. A step input is given to the system and its response is recorded. As an example, consider a system given by the following transfer function:

$$\frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1} \quad (1.33)$$

The step response shown in Figure 1.4 is generated using this transfer function with $K = 5 \text{ rad/V.s}$ and $\tau = 0.05 \text{ s}$.

The step input begins at time t_0 . The input signal has a minimum value of u_{min} and a maximum value of u_{max} . The resulting output signal is initially at y_0 . Once the step is applied, the output tries to follow it and eventually settles at its steady-state value y_{ss} . From the output and input signals, the steady-state gain is

$$K = \frac{\Delta y}{\Delta u} \quad (1.34)$$

where $\Delta y = y_{ss} - y_0$ and $\Delta u = u_{max} - u_{min}$. In order to find the model time constant, τ , we can first calculate where the output is supposed to be at the time constant from:

$$y(t_1) = 0.632y_{ss} + y_0 \quad (1.35)$$

Then, we can read the time t_1 that corresponds to $y(t_1)$ from the response data in Figure Figure 1.4. From the figure we can see that the time t_1 is equal to:

$$t_1 = t_0 + \tau \quad (1.36)$$

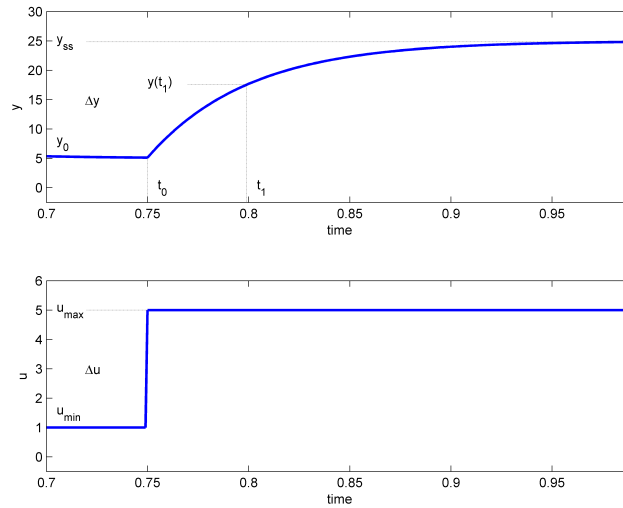


Figure 1.4: Input and output signal used in the bump test method

From this, the model time constant can be found as:

$$\tau = t_1 - t_0 \quad (1.37)$$

Going back to the Rotary Servo Base Unit system, a step input voltage with a time delay t_0 can be expressed as follows in the Laplace domain:

$$V_m(s) = \frac{A_v e^{(-s t_0)}}{s} \quad (1.38)$$

where A_v is the amplitude of the step and t_0 is the step time (i.e. the delay). If we substitute this input into the system transfer function given in Equation (Equation 1.1), we get:

$$\Omega_l(s) = \frac{K A_v e^{(-s t_0)}}{(\tau s + 1) s} \quad (1.39)$$

We can then find the Rotary Servo Base Unit load speed step response, $w_l(t)$, by taking inverse Laplace of this equation. Here we need to be careful with the time delay t_0 and note that the initial condition is $\omega_l(0^-) = \omega_l(t_0)$.

$$\omega_l(t) = K A_v \left(1 - e^{(-\frac{t-t_0}{\tau})} \right) + \omega_l(t_0) \quad (1.40)$$

2 Pre-Lab Questions

Before you start the lab experiments given in Section Section 3, you should study the background materials provided in Section 1 and work through the questions in this Section.

1. In Section 1.1.3 we obtained Equation 1.26 that described the dynamic behavior of the load shaft speed as a function of the motor input voltage. Starting from this equation, find the transfer function $\frac{\Omega_l(s)}{V_m(s)}$.
2. Express the steady-state gain (K) and the time constant (τ) of the process model Equation 1.1 in terms of the J_{eq} , $B_{eq,v}$, and A_m parameters.
3. Calculate the $B_{eq,v}$ and A_m model parameters using the system specifications given in Rotary Servo Base Unit User Manual. The parameters are to be calculated based on an Rotary Servo Base Unit in the high-gear configuration.
4. The load attached to the motor shaft includes a 24-tooth gear, two 72-tooth gears, and a single 120-tooth gear along with any other external load that is attached to the load shaft. Thus, for the gear moment of inertia J_g and the external load moment of inertia $J_{l,ext}$, the load inertia is $J_l = J_g + J_{l,ext}$. Using the specifications given in Rotary Servo Base Unit User Manual find the total moment of inertia J_g from the gears. **Hint:** Use the definition of moment of inertia for a disc $J_{disc} = \frac{mr^2}{2}$.
5. Assuming the disc load is attached to the load shaft, calculate the inertia of the disc load, J_d , and the total load moment of inertia acting on the motor shaft from the disc and gears, J_l .
6. Evaluate the equivalent moment of inertia J_{eq} . This is the total inertia from the motor, gears, and disc load. The moment of inertia of the DC motor can be found in the Rotary Servo Base Unit User Manual.
7. Calculate the steady-state model gain K and time constant τ . These are the *nominal model parameters* and will be used to compare with parameters that are later found experimentally.
8. Referring to Section 1.2.1, find the expression representing the time constant τ of the frequency response model given in Equation 1.31. Begin by evaluating the magnitude of the transfer function at the cutoff frequency ω_c .
9. Referring to Section 1.2.2, find the steady-state gain of the step response and compare it with Equation 1.34. **Hint:** The the steady-state value of the load shaft speed can be defined as $\omega_{l,ss} = \lim_{t \rightarrow \infty} \omega_l(t)$.
10. Evaluate the step response given in Equation 1.40 at $t = t_0 + \tau$ and compare it with Equation 1.34.

3 Lab Experiments

The main goal of this laboratory is to find a transfer function (model) that describes the rotary motion of the Rotary Servo Base Unit load shaft as a function of the input voltage. We can obtain this transfer function experimentally using one of the following two methods:

- Frequency response, or
- Step response

In this laboratory, first you will conduct two experiments exploring how these methods can be applied to a real system. Then, you will conduct a third experiment to fine tune the parameters of the transfer functions you obtained and to validate them.

Experimental Setup

The q_servo_modeling Simulink diagram shown in Figure 3.1 will be used to conduct the experiments. The Rotary Servo Base Unit subsystem contains QUARC® blocks that interface with the DC motor and sensors of the Rotary Servo Base Unit system. The Rotary Servo Base Unit Model uses a *Transfer Fcn* block from the SIMULINK® library to simulate the Rotary Servo Base Unit system. Thus, both the measured and simulated load shaft speed can be monitored simultaneously given an input voltage.

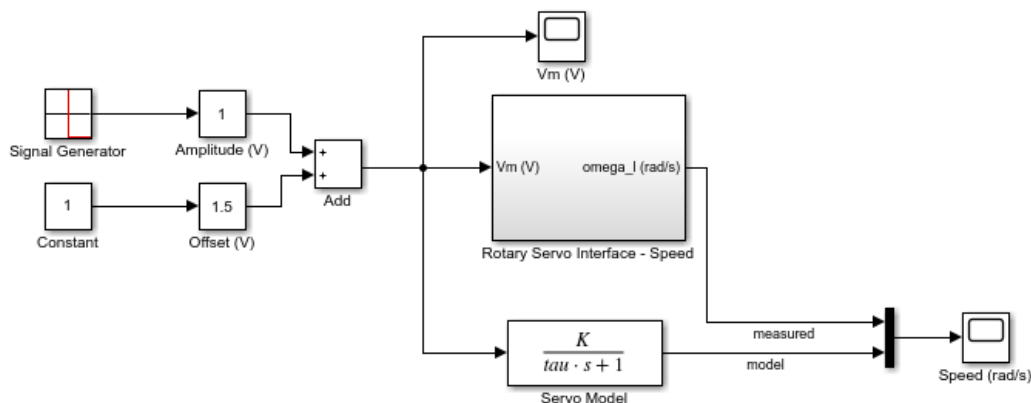


Figure 3.1: q_servo_modeling Simulink diagram used to model Rotary Servo Base Unit.

IMPORTANT: Before you can conduct these experiments, you need to make sure that the lab files are configured according to your Rotary Servo Base Unit setup. If they have not been configured already, then you need to go to Section 4.2 to configure the lab files first.

3.1 Frequency Response Experiment

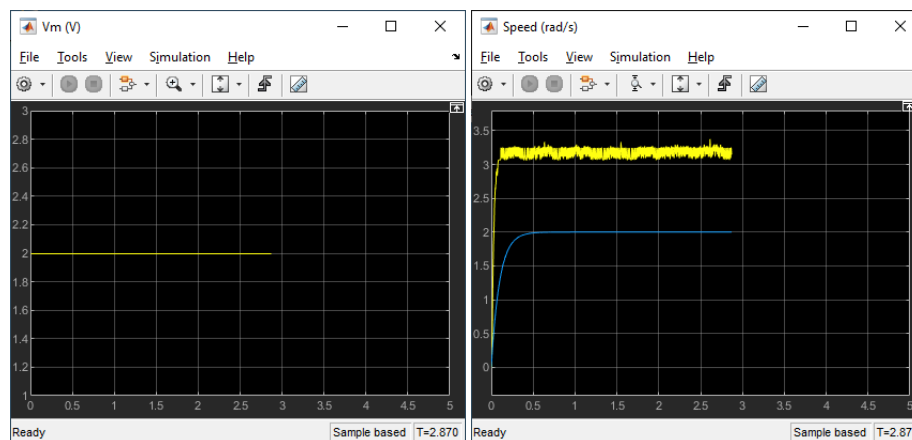
As explained in 1.2.1 earlier, the frequency response of a linear system can be obtained by providing a sine wave input signal to it and recording the resulting output sine wave from it. In this experiment, the input signal is the motor voltage and the output is the motor speed.

In this method, we keep the amplitude of the input sine wave constant but vary its frequency. At each frequency setting, we record the amplitude of the output sine wave. The ratio of the output and input amplitudes at a given frequency can then be used to create a Bode magnitude plot. Then, the transfer function for the system can be extracted from this Bode plot.

3.1.1 Steady-state gain

First, we need to find the steady-state gain of the system. This requires running the system with a constant input voltage. To create a 2V constant input voltage follow these steps:

1. In the **SIMULINK®** diagram, double-click on the *Signal Generator* block and ensure the following parameters are set:
 - Wave form: sine
 - Amplitude: 1.0
 - Frequency: 0.0
 - Units: Hertz
2. Set the *Amplitude (V)* slider gain to 0.
3. Set the *Offset (V)* block to 2.0 V.
4. Open the load shaft speed scope, *Speed (rad/s)*, and the motor input voltage scope, *V_m (V)*.
5. Click on QUARC | Build to compile the Simulink diagram.
6. Select QUARC | Start to run the controller. The Rotary Servo Base Unit unit should begin rotating in one direction. The scopes should be reading something similar to Figures Figure 3.2a and Figure 3.2b. Note that in the *Speed (rad/s)* scope, the yellow trace is the measured speed while the blue trace is the simulated speed (generated by Servo Model block).



(a) Constant input motor voltage.

(b) Load shaft speed response

Figure 3.2: Load shaft speed response to a constant input

7. Measure the speed of the load shaft and enter the measurement in Table Table 3.1 below under the $f = 0$ Hz row.

Hint: The measurement can be done directly from the scope using the *Cursor Measurements* tool. Alternatively, you can use **MATLAB®** commands to find the maximum load speed using the saved *w_l* variable. When the controller is stopped, the *w_l* (rad/s) scope saves the last 10 seconds of response data to the **MATLAB®** workspace in the *w_l* parameter. It has the following structure: *w_l*(:,1) is the time vector, *w_l*(:,2) is the measured speed, and *w_l*(:,3) is the simulated speed.

8. Calculate the steady-state gain both in linear and decibel (dB) units as explained in 1.2.1. Enter the resulting numerical value in the $f = 0$ Hz row of Table Table 3.1. Also, enter its non-decibel value in Table 3.2 in Section 3.4.

3.1.2 Gain at varying frequencies

In this part of the experiment, we will send an input sine wave at a certain frequency to the system and record the amplitude of the output signal. We will then increment the frequency and repeat the same observation.

To create the input sine wave:

1. In the **SIMULINK**® diagram, double-click on the *Signal Generator* block and ensure the following parameters are set:
 - Wave form: sine
 - Amplitude: 1.0
 - Frequency: 1.0
 - Units: Hertz
2. Set the *Amplitude (V)* slider gain to 2.0 V.
3. Set the *Offset (V)* block to 0 V.
4. Run the q_servo_modeling QUARC controller to apply the 2V 1 Hz sine wave to the Rotary Servo Base Unit.
5. The Rotary Servo Base Unit unit should begin rotating smoothly back and forth and the scopes should be reading a response similar to Figures Figure 3.3a and Figure 3.3b.

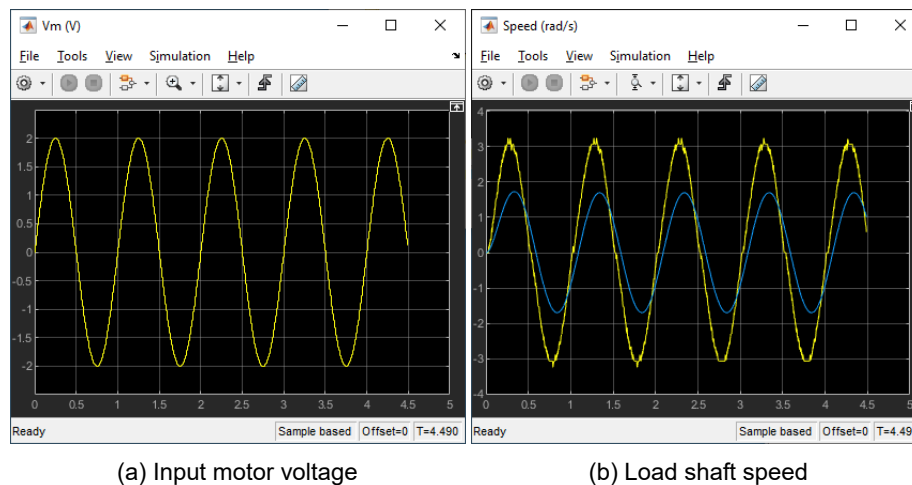


Figure 3.3: Load shaft speed sine wave response

6. Measure the maximum positive speed of the load shaft at $f = 1.0$ Hz input and enter it in Table 3.1 below. As before, this measurement can be done directly from the scope using the *Cursor Measurements* tool or you can use **MATLAB**® commands to find the maximum load speed using the saved *wl* variable.
7. Calculate the gain of the system (in both linear and dB units) and enter the results in Table 3.1.
8. Increase the frequency to $f = 2.0$ Hz by adjusting the frequency parameter in the *Signal Generator* block. Measure the maximum load speed and calculate the gain. Repeat this step for each of the frequency settings in Table 3.1.
9. Using the **MATLAB**® *plot* command and the data collected in Table 3.1, generate a Bode magnitude plot. Make sure the amplitude and frequency scales are in decibels. When making the Bode plot, ignore the $f = 0$ Hz entry as the logarithm of 0 is not defined.
10. Calculate the time constant $\tau_{e,f}$ using the obtained Bode plot by finding the cutoff frequency. Label the Bode plot with the -3 dB gain and the cutoff frequency. Enter the resulting time constant in Table 3.2. **Hint:** Use the *Data Tips* tool to obtain values from the **MATLAB**® Figure.

- Click the Stop button on the **SIMULINK®** diagram toolbar (or select QUARC | Stop from the menu) to stop the experiment.
- Turn off the power to the amplifier if no more experiments will be performed on the Rotary Servo Base Unit in this session.

f (Hz)	Amplitude (V)	Max Load Speed (rad/s)	Gain: G(ω) (rad/s/V)	Gain: G(ω) (rad/s/V, dB)
0.0	2.0			
1.0	2.0			
2.0	2.0			
3.0	2.0			
4.0	2.0			
5.0	2.0			
6.0	2.0			
7.0	2.0			
8.0	2.0			

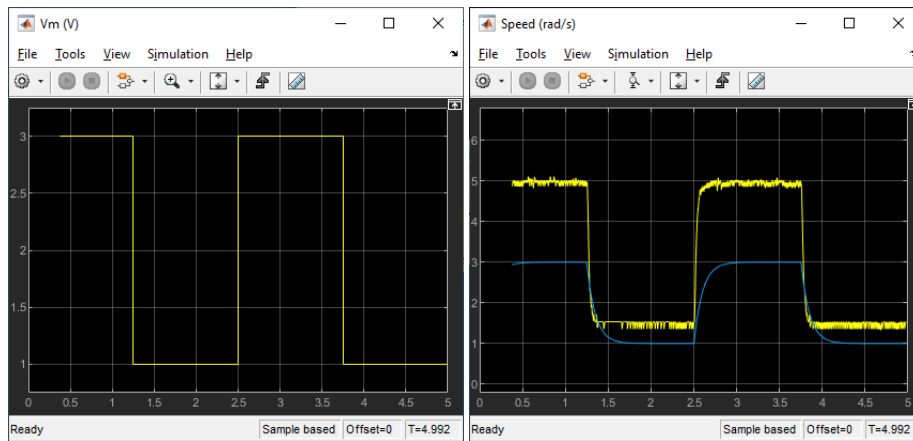
Table 3.1: Collected frequency response data.

3.2 Step Response Experiment

In this method, a step input is given to the Rotary Servo Base Unit and the corresponding load shaft response is recorded. Using the saved response, the model parameters can then be found as discussed in Section 1.2.2.

To create the step input:

- Double-click on the *Signal Generator* block and ensure the following parameters are set:
 - Wave form: square
 - Amplitude: 1.0
 - Frequency: 0.4
 - Units: Hertz
- Set the Amplitude (V) gain block to 1.0 V.
- Set the Offset (V) gain block to 2.0 V.
- Open the load shaft speed scope, *Speed (rad/s)*, and the motor input voltage cope, *V_m (V)*.
- Click on QUARC | Build to compile the Simulink diagram.
- Select QUARC | Start to run the controller. The gears on the Rotary Servo Base Unit should be rotating in the same direction and alternating between low and high speeds. The response in the scopes should be similar to Figures Figure 3.4a and Figure 3.4b.
- Plot the response in **MATLAB®**. Recall that the maximum load speed is saved in the **MATLAB®** workspace under the *wl* variable.
- Find the steady-state gain using the measured step response and enter it in Table 3.2. *Hint: Use the **MATLAB®** `ginput` command to measure points off the plot.*
- Find the time constant from the obtained response and enter the result in Table 3.2.



(a) Square input motor voltage.

(b) Load shaft speed step response.

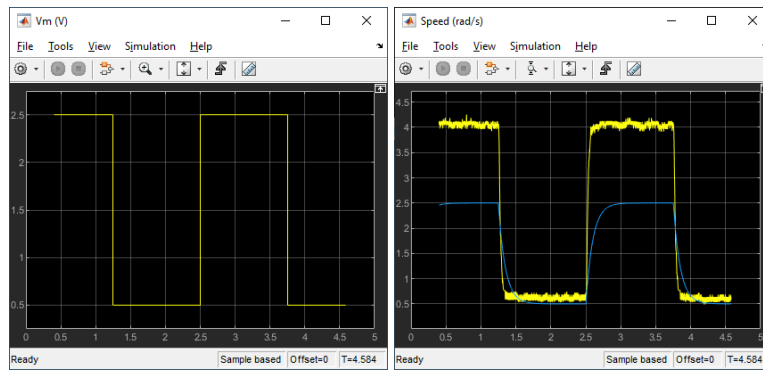
10. Click the Stop button on the **SIMULINK®** diagram toolbar (or select QUARC | Stop from the menu) to stop the experiment.
11. Turn off the power to the amplifier if no more experiments will be performed on the Rotary Servo Base Unit in this session.

3.3 Model Validation Experiment

In this experiment, you will adjust the model parameters you found in the previous experiments to tune the transfer function. Our goal is to match the simulated system response with the parameters you found as closely as possible to the response of the actual system.

To create a step input:

1. Double-click on the *Signal Generator* block and ensure the following parameters are set:
 - Wave form: square
 - Amplitude: 1.0
 - Frequency: 0.4
 - Units: Hertz
2. Set the *Amplitude (V)* slider gain to 1.0 V.
3. Set the *Offset (V)* block to 1.5 V.
4. Open the load shaft speed scope, w_l (rad/s), and the motor input voltage scope, V_m (V).
5. Click on QUARC | Build to compile the **SIMULINK®** diagram.
6. Select QUARC | Start to run the controller. The gears on the Rotary Servo Base Unit should be rotating in the same direction and alternating between low and high speeds and the scopes should be as shown in figures Figure 3.5a and Figure 3.5b. Recall that the yellow trace is the measured load shaft rate and the purple trace is the simulated trace. By default, the steady-state gain and the time constant of the transfer function used in simulation are set to: $K = 1 \text{ rad/s/V}$ and $\tau = 0.1 \text{ s}$. These model parameters do not accurately represent the system.
7. Enter the command $K = 1.25$ in the **MATLAB®** Command Window.
8. Update the parameters used by the Transfer Function block in the simulation by selecting Update Diagram in the q_servo_modeling **SIMULINK®** diagram and observe how the simulation changes.
9. Enter the command $\tau = 0.2$ in the **MATLAB®** Command Window.



(a) Input square voltage.

(b) Speed step response.

Figure 3.5: Simulation done with default model parameters: $K = 1$ and $\tau = 0.1$.

10. Update the simulation again by selecting Update Diagram and observe how the simulation changes.
11. Vary the gain and time constant model parameters. How do the gain and the time constant affect the system response?
12. Enter the nominal values, K and τ , that were found in Section 2 in the **MATLAB**® Command Window. Update the parameters and examine how well the simulated response matches the measured one.
13. If the calculations were done properly, then the model should represent the actual system quite well. However, there are always some differences between each servo unit and, as a result, the model can always be tuned to match the system better. Try varying the model parameters until the simulated trace matches the measured response better. Enter these tuned values under the Model Validation section of Table 3.2.
14. Provide two reasons why the nominal model does not represent the Rotary Servo Base Unit with better accuracy.
15. Create a **MATLAB**® figure that shows the measured and simulated response of the model validation response using the *tuned* model parameters of K and τ . This can be done by manually changing the Servo Model Transfer Function block parameters or changing the K and τ parameters in the **MATLAB**® Command Window and going to Simulation | Update Diagram in the **SIMULINK**® model.
16. Explain how well the nominal model, the frequency response model, and the bump test model represent the Rotary Servo Base Unit system.
17. Click the Stop button on the **SIMULINK**® diagram toolbar (or select QUARC | Stop from the menu) to stop the experiment.
18. Turn off the power to the amplifier if no more experiments will be performed on the Rotary Servo Base Unit in this session.

3.4 Results

1. Fill out Table 3.2 below, with your results.

From Section	Description	Symbol	Value	Unit
Section 2	Nominal Values			
	Open-Loop Steady-State Gain	K		rad/(V.s)
	Open-Loop Time Constant	τ		s
Section 3.1.1	Frequency Response Exp.			
	Open-Loop Steady-State Gain	$K_{e,f}$		rad/(V.s)
	Open-Loop Time Constant	$\tau_{e,f}$		s
Section 3.2	Bump Test Exp.			
	Open-Loop Steady-State Gain	$K_{e,b}$		rad/(V.s)
	Open-Loop Time Constant	$\tau_{e,b}$		s
Section 3.3	Model Validation			
	Open-Loop Steady-State Gain	$K_{e,v}$		rad/(V.s)
	Open-Loop Time Constant	$\tau_{e,v}$		s

Table 3.2: Summary of results for the Rotary Servo Base Unit Modeling laboratory.

4 File Description and Configuration

4.1 Overview of Files

File Name	Description
Rotary Servo Base Unit - Modeling Workbook (Student).pdf	This laboratory guide contains pre-lab and in-lab exercises demonstrating how to model the Quanser Rotary Servo Base Unit rotary plant. The in-lab exercises are explained using the QUARC software.
setup_servo_modeling.m	The main MATLAB® script that sets the Rotary Servo Base Unit motor and sensor parameters. Run this file only to setup the laboratory.
config_servo.m	Returns the configuration-based Rotary Servo Base Unit model specifications R_m , k_t , k_m , K_g , η_{a_g} , B_{eq} , J_{eq} , and η_{a_m} , the sensor calibration constants K_{POT} and K_{ENC} , and the amplifier limits V_{MAX_AMP} and I_{MAX_AMP} .
calc_conversion_constants.m	Returns various conversions factors.
q_servo_modeling	SIMULINK® file that implements the open-loop controller for the Rotary Servo Base Unit system using QUARC.

Table 4.1: Files supplied with the Rotary Servo Base Unit Modeling laboratory.

4.2 Configuring the Rotary Servo Base Unit and the Lab Files

Before beginning the lab exercises the Rotary Servo Base Unit device, the q_servo_modeling **SIMULINK®** diagram, and the setup_servo_modeling.m script must be configured.

Follow these steps to get the system ready for this lab:

1. Set up the Rotary Servo Base Unit in the high-gear configuration and with the disc load as described in Rotary Servo Base Unit User Manual.
2. Load the **MATLAB®** software.
3. Browse through the Current Directory window in **MATLAB®** and find the folder that contains the Rotary Servo Base Unit modeling files, e.g. q_servo_modeling.
4. Double-click on the q_servo_modeling file to open the **SIMULINK®** diagram shown in Figure 3.1.
5. **Configure DAQ:** Double-click on the HIL Initialize block in the **SIMULINK®** diagram and ensure it is configured for the DAQ device that is installed in your system (e.g. Q2-USB). See **QUARC®** documentation for more information on configuring the HIL Initialize block.
6. Go to the *Current Directory* window and double-click on the setup_servo_modeling.m file to open the setup script for the q_servo_modeling **SIMULINK®** model.
7. **Configure setup script:** Ensure the script is setup to match the configuration of your actual Rotary Servo Base Unit device. For example, the script given below is setup for an Rotary Servo Base Unit plant in the high-gear configuration mounted with a disc load and it is actuated using the Quanser VoltPAQ-X1 power amplifier

with a amplifier gain of 1. See Rotary Servo Base Unit User Manual for more information on Rotary Servo Base Unit plant options and corresponding accessories.

8. Set the MODELING_TYPE is set to 'MANUAL'.

```
%% Rotary Servo Configuration
% External Gear Configuration: set to 'HIGH' or 'LOW'
EXT_GEAR_CONFIG = 'HIGH';
% Type of Load: set to 'NONE', 'DISC', or 'BAR'
LOAD_TYPE = 'DISC';
% Amplifier Gain: set VoltPAQ amplifier gain to 1
K_AMP = 1;
% Power Amplifier Type: set to 'VoltPAQ', 'UPM_1503', or 'UPM_2405'
AMP_TYPE = 'VoltPaq';
%
%% Lab Configuration
% Type of Controller: set it to 'AUTO', 'MANUAL'
% MODELING_TYPE = 'AUTO';
MODELING_TYPE = 'MANUAL';
%
```

9. Run the script. The messages shown below should be generated in the **MATLAB®** Command Window. *These are default model parameters and do not accurately represent the Rotary Servo Base Unit system.*

```
Calculated servo model parameter:
K = 1 rad/s/V
tau = 0.1 s
```

5 Lab Report

When you prepare your lab report, you can follow the outline given in Section 5.1 to build the *content* of your report. Also, in Section 5.2 you can find some basic tips for the *format* of your report.

5.1 Template for Content

I. PROCEDURE

I.1. Frequency Response Experiment

1. Briefly describe the main goal of this experiment and the procedure.
 - Briefly describe the experimental procedure (Section 3.1.1), *Steady-state gain*
 - Briefly describe the experimental procedure (Section 3.1.2), *Gain at varying frequencies*

I.2. Bump Test Experiment

1. Briefly describe the main goal of this experiment and the experimental procedure (Section 3.2).

I.3. Model Validation Experiment

1. Briefly describe the main goal of this experiment and the experimental procedure (Section 3.3).

II. RESULTS

Do not interpret or analyze the data in this section. Just provide the results.

1. Bode plot from step 9 in Section 3.1.2, *Gain at varying frequencies*.
2. Response plot from step 7 in Section 3.2, *Bump Test Experiment*.
3. Response plot from step 15 in Section 3.3, *Model Validation Experiment*.
4. Provide data collected in this laboratory (from Table Table 3.1).

III. ANALYSIS

Provide details of your calculations (methods used) for analysis for each of the following:

III.1. Frequency Response Experiment

1. Step 8 in Section 3.1.1, *Steady-state gain*.
2. Step 10 in Section 3.1.2, *Gain at varying frequencies*.

III.2. Bump Test Experiment

1. Steps 8 and 9 in Section 3.2.

IV. CONCLUSIONS

Interpret your results to arrive at logical conclusions.

1. Steps 11, 14, and 16 in Section 3.3.

5.2 Tips for Report Format

PROFESSIONAL APPEARANCE

- Has cover page with all necessary details (title, course, student name(s), etc.)
- Each of the required sections is completed (Procedure, Results, Analysis and Conclusions).
- Typed.
- All grammar/spelling correct.
- Report layout is neat.
- Does not exceed specified maximum page limit, if any.
- Pages are numbered.
- Equations are consecutively numbered.
- Figures are numbered, axes have labels, each figure has a descriptive caption.
- Tables are numbered, they include labels, each table has a descriptive caption.
- Data are presented in a useful format (graphs, numerical, table, charts, diagrams).
- No hand drawn sketches/diagrams.
- References are cited using correct format.

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Waste Electrical and Electronic Equipment (WEEE)



This symbol indicates that waste products must be disposed of separately from municipal household waste, according to Directive 2002/96/EC of the European Parliament and the Council on waste electrical and electronic equipment (WEEE). All products at the end of their life cycle must be sent to a WEEE collection and recycling center. Proper WEEE disposal reduces the environmental impact and the risk to human health due to potentially hazardous substances used in such equipment. Your cooperation in proper WEEE disposal will contribute to the effective usage of natural resources. For information about the available collection and recycling scheme in a particular country, go to ni.com/citizenship/weee.

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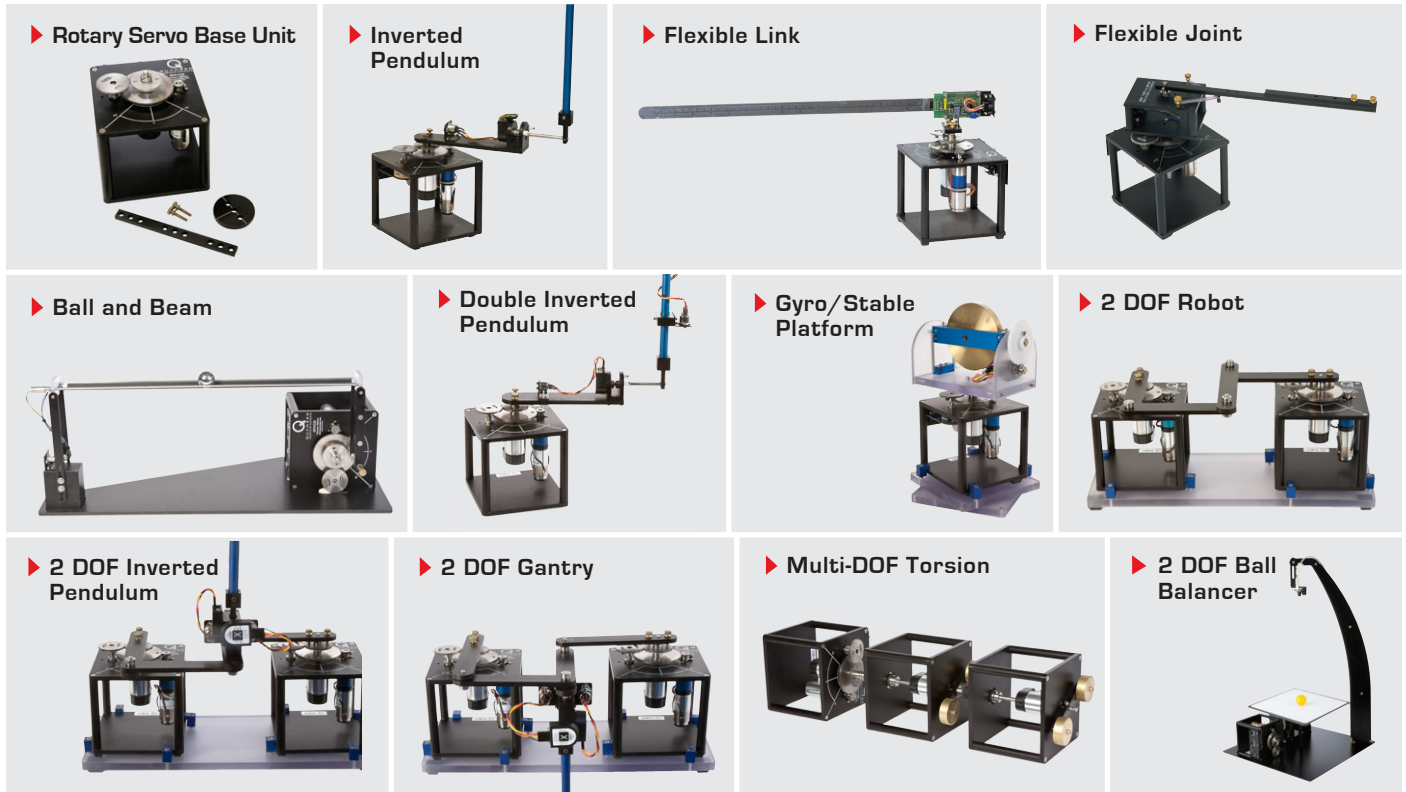
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This product meets the essential requirements of applicable European Directives as follows:

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