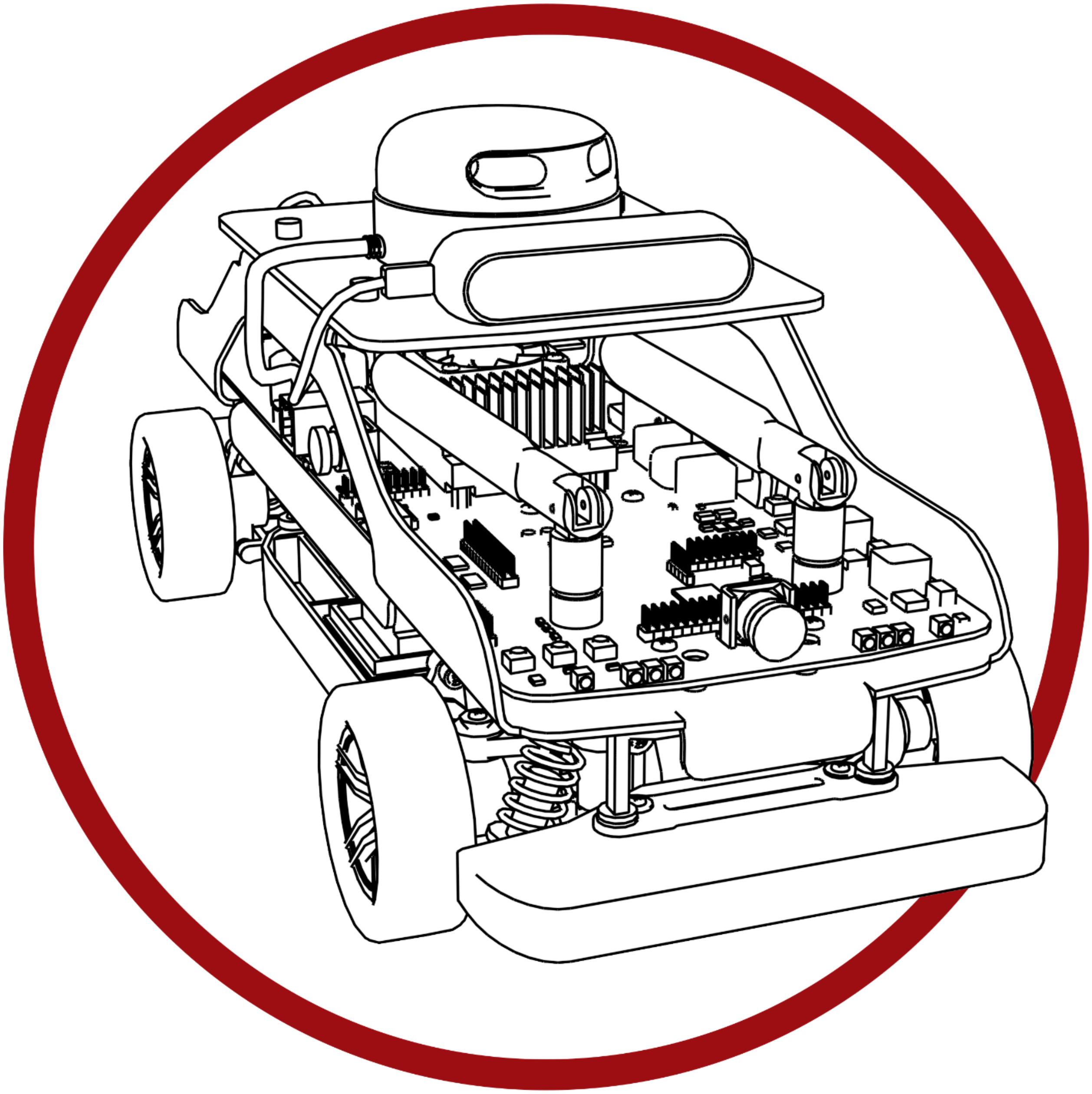
Self-Driving Car Research Studio



Sensor Fusion via Complementary Filters

V 1.1 (November 2020)

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Basics

Low pass filters

These filters remove noise with frequencies above a threshold ω_n, also known as the filter frequency in rad/s. It allows signal frequencies below the threshold through, hence the name - low pass. It can be represented using a transfer function

$$G(s) = \frac{\omega_n}{s + \omega_n} \tag{1}$$

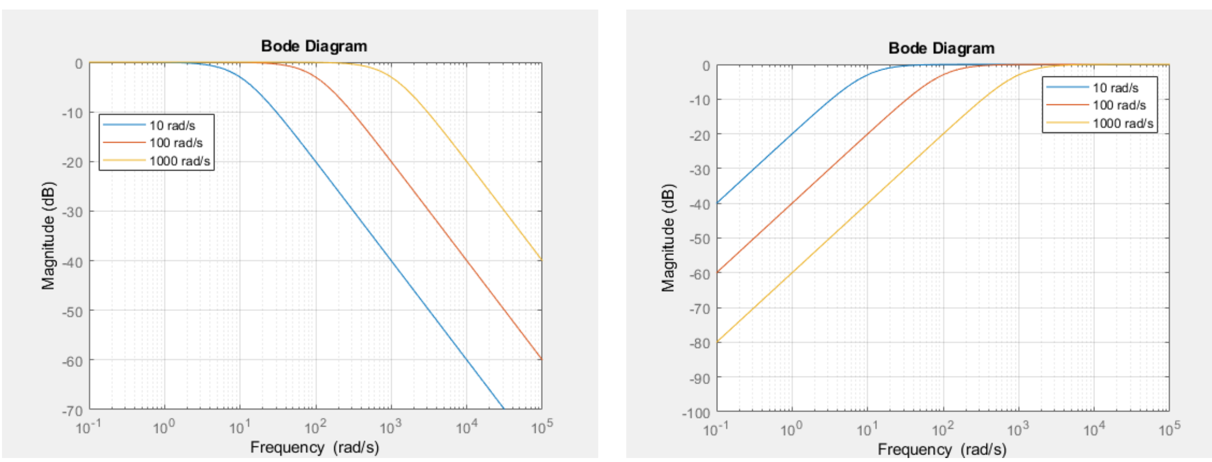
As shown in figure 1a, the power level of the signal frequencies below the filter frequency is mostly 0 dB, which signifies that the power level remains unchanged. Above the filter frequency, a negative power level signifies power reduction.

High pass filters

These filters remove noise with frequencies below a threshold ω_n, also known as the filter frequency in rad/s. It allows signal frequencies above the threshold through, hence the name - high pass. It can be represented using a transfer function

$$H(s) = \frac{s}{s + \omega_n} \tag{2}$$

As shown in figure 1b, the power level of the signal frequencies above the filter frequency is mostly 0 dB, which signifies that the power level remains unchanged. Below the filter frequency, a negative power level signifies power reduction.



a. Low pass filters

b. High pass filters

Figure 1: Bode plot showing magnitude of signal frequencies

Complementary Filters

Consider figure 2, that shows the bode magnitude plot of a low pass and high pass filter with the same frequency (in this case, 100 rad/s as an example).

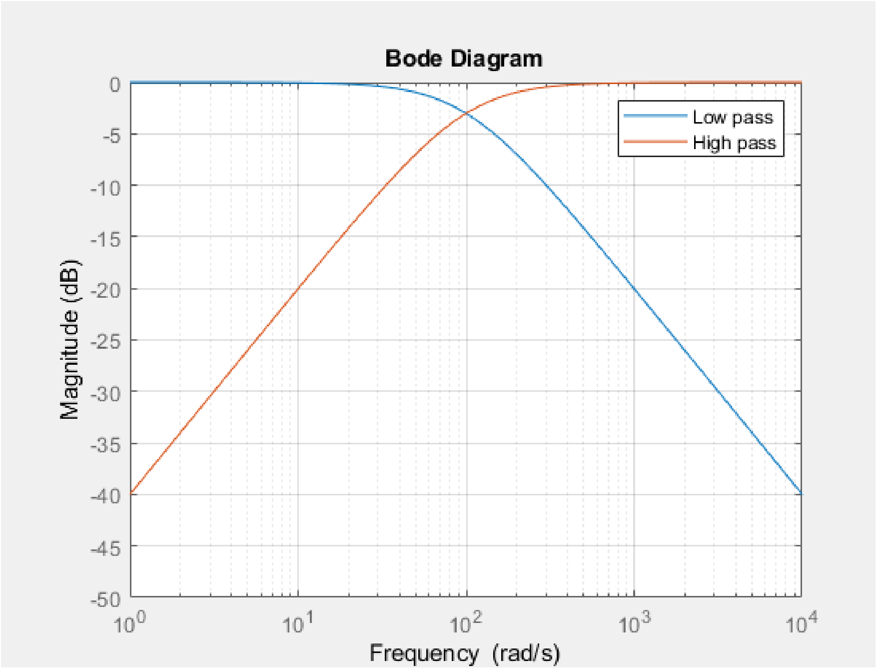


Figure 2: Bode plot showing a low and high pass filter with the same frequency.

Figure 2 demonstrates that if we pass a signal through a low-pass and high-pass filter in parallel, the sum of the two filter outputs should have a magnitude of 0 db almost everywhere. As such, the two filters represented by equations 1 and 2 are 'complementary' in nature. This can also be verified as,

$$\frac{\omega_n}{s+\omega_n}\,x \;+\; \frac{s}{s+\omega_n}\,x = \left(\frac{\omega_n}{s+\omega_n} + \frac{s}{s+\omega_n}\right) x \;=\; x \tag{3}$$

Thus, given two filters $H(s)$ and $G(s)$, they are said to be complementary, if they satisfy the following equation

$$H(s) + G(s) = 1 \tag{4}$$

Sensor Fusion

Consider a rate signal $\dot{x}$ (such as gyroscopic yaw rates), integrated to estimate the signal x_e (such as yaw angle). This signal is prone to integration drift due to noise in the rate signal. This is shown in a block diagram format (via Simulink as an example) in figure 3 below.

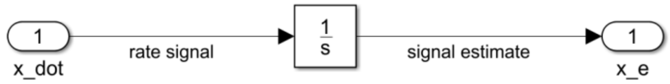


Figure 3. Rate signal integration to estimate signal value as a basic example.

We could correct the rate signal itself based on the drift in the signal estimate. If the signal estimate drift is positive, we could reduce the signal rate being integrated and vice versa, as shown in figure 4.

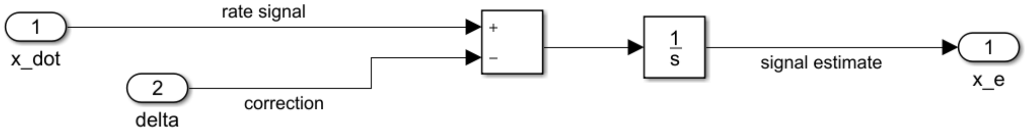


Figure 4: Correcting the rate signal before integration to remove drift.

It is desirable that this correction be programmatically estimated based on the drift with respect to a reference signal. For example, with drones, we have a gyroscopic pitch rate that could be integrated to achieve the pitch angle estimate, but we also have a noisy and lower rate pitch approximation based on the accelerometer that could be used to correct the gyroscopic pitch rate. Given this, a proportional-integral compensator on the error could provide the correction, as shown in figure 5.

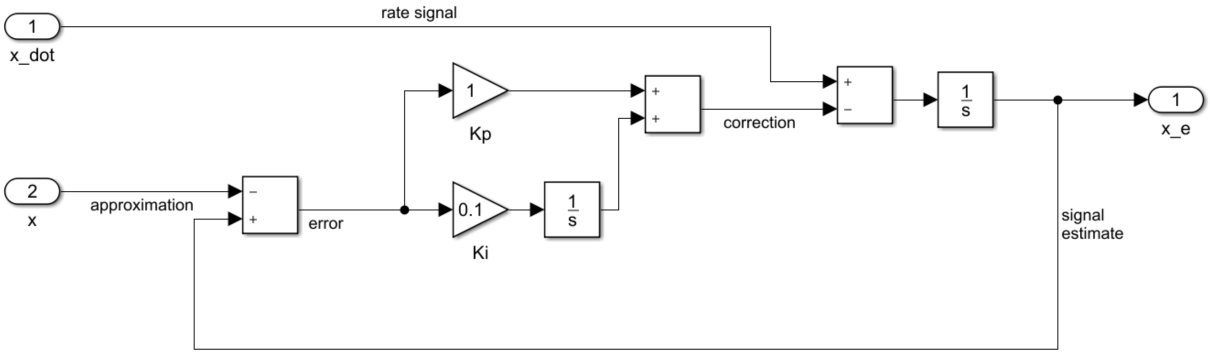


Figure 5: Correcting rate signal before integration using the error between the signal estimate and approximation

In this sense, x is typically an approximation that is available at a slower sample rate and contains undesirable higher frequency noise. The rate signal $\dot{x}$ is the derivative of the approximate x available through an independent sensor and measurement. It is typically sampled at higher rates but captures the motion dynamics of the signal in consideration.

Deriving the transfer function for x_e based on the inputs $\dot{x}$ and x provides,

$$x_e(s) = \frac{s}{s^2 + K_p s + K_i}\dot{x}(s) + \frac{K_p s + K_i}{s^2 + K_p s + K_i}x(s) \tag{5}$$

In the equation above, we are comparing $\dot{x}$ with x, two signals that have different physical dimensions and meaning. Adding an s term to the numerator and denominator of the first transfer function yields,

$$x_e(s) = \frac{s^2}{s^2 + K_p s + K_i}\frac{\dot{x}(s)}{s} + \frac{K_p s + K_i}{s^2 + K_p s + K_i}x(s) \tag{6}$$

In this case, we compare the integral of $\dot{x}$ with x, which is physically comparable. Note the two transfer functions,

$$\begin{aligned} H(s) &= \frac{s^2}{s^2 + K_p s + K_i} \\ G(s) &= \frac{K_p s + K_i}{s^2 + K_p s + K_i} \end{aligned} \tag{7}$$

These are second-order versions of high-pass $H(s)$ and low-pass $G(s)$ filters respectively. Verify that their sum is indeed unity. Thus, complementary filters for sensor fusion involves a high-pass filter on the integral of a rate signal $\dot{x}$ to capture the fast frequency motion dynamics, while passing the low-frequency corrections from an approximation x to achieve a signal estimate that does not drift and has inherently less noise.